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# MATHEMATICS today

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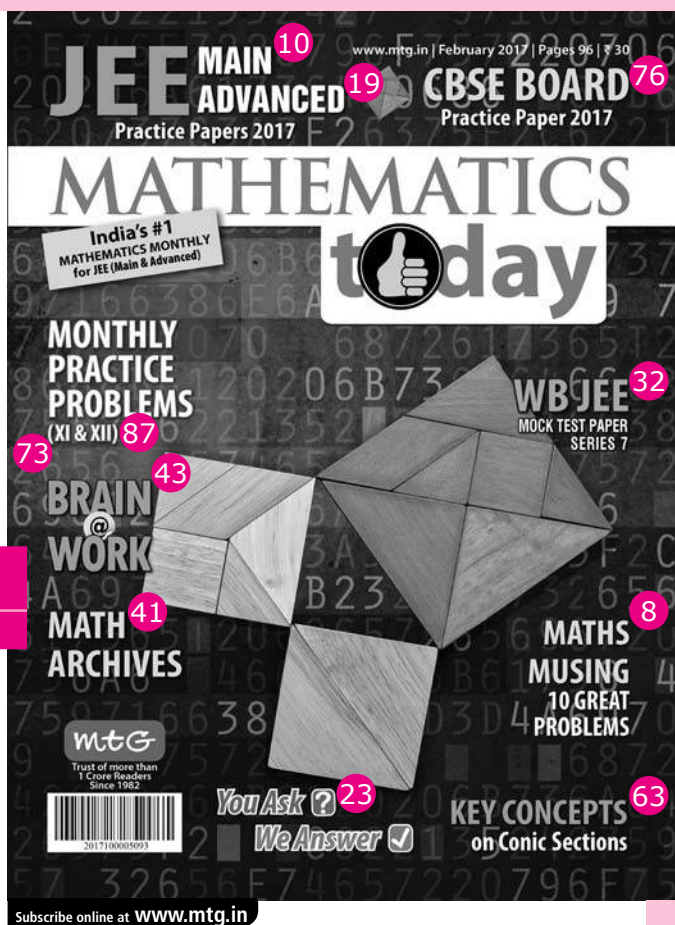
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# MATHS MUSING

**M**aths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

## PROBLEM Set 170

### JEE MAIN

- If the tangent at the point  $(a, b)$  on the curve  $x^3 + y^3 = a^3 + b^3$  meets the curve again at the point  $(p, q)$ , then  
(a)  $ap + bq + ab = 0$  (b)  $bp + aq + ab = 0$   
(c)  $ap + bq - pq = 0$  (d)  $bp + aq - pq = 0$
- Let  $\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} \sqrt{3} + 1 & 1 - \sqrt{3} \\ \sqrt{3} - 1 & 1 + \sqrt{3} \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix}$ ,  $n \in \mathbb{N}$ .  
If  $a_1 = b_1 = 1$ , then  $a_{22} =$   
(a)  $2^{31}$  (b)  $2^{32}$  (c)  $2^{33}$  (d)  $3^{33}$
- In a triangle  $ABC$ , if  $\tan A : \tan B : \tan C = 2 : 3 : 4$ , then  $\frac{\Delta}{R^2} =$   
(a)  $\frac{9\sqrt{6}}{35}$  (b)  $\frac{18\sqrt{6}}{35}$  (c)  $\frac{36\sqrt{2}}{35}$  (d)  $\frac{18\sqrt{3}}{35}$
- If  $z = \frac{\sqrt{3} - i}{2}$ , then  $(i^{101} + z^{101})^{103} =$   
(a)  $z$  (b)  $\bar{z}$  (c)  $iz$  (d)  $i\bar{z}$
- Let  $m$  be the number of 5-element subsets that can be chosen from the set of the first 15 natural numbers so that at least two of the five numbers are consecutive. The sum of the digits of  $m$  is  
(a) 11 (b) 12 (c) 13 (d) 14

### JEE ADVANCED

- Let  $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ . A value of  $\lambda$ , such that  $AX = \lambda X$ , where  $X$  is a non-zero column vector, is  
(a) 0 (b) 1 (c) 2 (d) 3

### COMPREHENSION

The parametric equations of a parabola are  $x = u \cos \alpha t$ ,  $y = u \sin \alpha t - \frac{1}{2}gt^2$ ,  $t$  is a parameter.

- Its latus rectum is  
(a)  $\frac{2u^2}{g} \sin^2 \alpha$  (b)  $\frac{2u^2}{g} \cos^2 \alpha$

(c)  $\frac{u^2}{2g} \sin^2 \alpha$  (d)  $\frac{u^2}{2g} \cos^2 \alpha$

- Its focus is  
(a)  $\left( \frac{u^2}{2g} \sin 2\alpha, \frac{u^2}{2g} \cos 2\alpha \right)$   
(b)  $\left( \frac{u^2}{2g} \sin 2\alpha, \frac{-u^2}{2g} \cos 2\alpha \right)$   
(c)  $\left( \frac{u^2}{2g} \sin^2 \alpha, \frac{-u^2}{2g} \cos^2 \alpha \right)$   
(d)  $\left( \frac{u^2}{2g} \cos^2 \alpha, \frac{-u^2}{2g} \sin^2 \alpha \right)$

### INTEGER MATCH

- $I$  is the incentre of the triangle  $ABC$ .  $AI, BI, CI$  when produced meet the opposite sides at  $D, E, F$  respectively. Then the value of  $\frac{AI}{ID} \cdot \frac{BI}{IE} \cdot \frac{CI}{IF} - \left( \frac{AI}{ID} + \frac{BI}{IE} + \frac{CI}{IF} \right)$  is

### MATRIX MATCH

- List-I contains  $S$  and List-II gives last digit of  $S$ .

	List-I		List-II
P.	$S = \sum_{n=1}^{11} (2n-1)^2$	1.	0
Q.	$S = \sum_{n=1}^{10} (2n-1)^3$	2.	1
R.	$S = \sum_{n=1}^{18} (2n-1)^2 (-1)^n$	3.	5
S.	$S = \sum_{n=1}^{15} (2n-1)^3 (-1)^{n-1}$	4.	8

P	Q	R	S
(a) 4	2	1	3
(b) 4	2	3	1
(c) 2	1	4	3
(d) 1	2	3	4

See Solution Set of Maths Musing 169 on page no. 89

# KNOWLEDGE SERIES

## (for JEE / Olympiad Aspirants)

Myth	Reality
More focus on two subjects can improve my rank in JEE	JEE Paper contains basic & conceptual question, skipping one subject can reduce your marks which could result in low ranking and not getting cutoff marks. Suggestion-Focus on all subjects (PCM) & try to solve basic and conceptual question of all topics.

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# PRACTICE PAPER 2017 JEE MAIN

**Exam Dates**  
OFFLINE : 2<sup>nd</sup> April  
ONLINE : 8<sup>th</sup> & 9<sup>th</sup> April

- Three numbers are chosen at random from 1, 2, ..., 30. What is the probability that they will form a G.P?  
(a)  $\frac{12}{4060}$  (b)  $\frac{10}{4060}$  (c)  $\frac{19}{4060}$  (d)  $\frac{20}{4060}$
- If  $n$  balls are distributed into  $m$  boxes, so that each ball is equally likely to fall in any box, the probability that a specified box will contain  $r$  balls is  
(a)  $\frac{{}^nC_r(m-1)^{n-r}}{{}^nC_m}$  (b)  $\frac{{}^nP_r(m-1)^{n-r}}{{}^nC_m}$   
(c)  $\frac{{}^nC_r(m-1)^{n-r}}{m^n}$  (d)  $\frac{{}^nC_r(m-1)^{n-r}}{n^m}$
- The probability that at least one of the events  $A$  and  $B$  occur is 0.4. If  $A$  and  $B$  occur simultaneously with probability 0.1, then probability of  $A^C$  or  $B^C$  is equal to  
(a) 1.2 (b) 1.4 (c) 1.5 (d) 1.8
- In an attempt to land an unmanned rocket on the moon, the probability of a successful landing is 0.4 and the probability of monitoring system giving the correct information of landing is 0.9 in either case. The probability of a successful landing, it being known that the monitoring system indicated it correctly  
(a)  $\frac{2}{5}$  (b)  $\frac{4}{25}$  (c)  $\frac{9}{10}$  (d)  $\frac{6}{7}$
- Each of  $N + 1$  identical urns marked 0, 1, 2, ...,  $N$  contains  $N$  balls so that the  $i^{\text{th}}$  urn contains  $i$  black and  $N - i$  white balls ( $0 \leq i \leq N$ ). An urn is chosen at random and  $n$  balls are drawn from it one after another with replacement. If all the  $n$  balls turn out to be black, the probability that the next ball will also be black [assume that  $N$  is large] is  
(a)  $\frac{n+1}{n+2}$  (b)  $\frac{n-1}{n+1}$  (c)  $\frac{n}{n+1}$  (d)  $\frac{n}{n+2}$
- The probability that a teacher will give a surprise test during any class meeting is  $\frac{3}{5}$ . If a student is absent on two days, the probability that he will miss at least one test is  
(a)  $\frac{2}{5}$  (b)  $\frac{3}{5}$  (c)  $\frac{20}{25}$  (d)  $\frac{21}{25}$
- If two independent events  $A$  and  $B$  are such that  $P(A \cap B^C) = \frac{3}{25}$  and  $P(A^C \cap B) = \frac{8}{25}$  and  $P(A) > \frac{1}{2}$ , the value of  $P(A) + P(B) =$   
(a)  $\frac{8}{5}$  (b)  $\frac{4}{5}$   
(c)  $\frac{7}{5}$  (d) None of these
- If two events  $A$  and  $B$  are such that  $P(A^C) = 0.3$ ,  $P(B) = 0.4$ ,  $P(AB^C) = 0.5$ , then  $P(B|A \cup B^C)$  is  
(a)  $\frac{1}{4}$  (b)  $\frac{1}{5}$  (c)  $\frac{4}{5}$  (d)  $\frac{1}{2}$
- The independent probabilities that  $A$ ,  $B$ ,  $C$  can solve the problem are  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$  respectively. The probability that just two of them only solve the problem is  
(a)  $\frac{3}{4}$  (b)  $\frac{2}{3}$   
(c)  $\frac{1}{4}$  (d) None of these
- Bag  $A$  contains 2 white and 3 red balls and bag  $B$  contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. The probability that it was drawn from bag  $B$  is  
(a)  $\frac{25}{52}$  (b)  $\frac{13}{27}$  (c)  $\frac{8}{17}$  (d)  $\frac{25}{51}$
- The set of equations  
 $\lambda x - y + (\cos \theta)z = 0$   
 $3x + y + 2z = 0$   
 $(\cos \theta)x + y + 2z = 0$   
 $0 \leq \theta < 2\pi$ , has non-trivial solutions  
(a) for no value of  $\lambda$  and  $\theta$   
(b) for all values of  $\lambda$  and  $\theta$   
(c) for all values of  $\lambda$  and only two values of  $\theta$   
(d) for only one value of  $\lambda$  and all values of  $\theta$ .



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12. Orthogonal trajectories of family of parabolas  $y^2 = 4a(x + a)$  where 'a' is an arbitrary constant is

(a)  $ax^2 = 4cy$   
 (b)  $x^2 + y^2 = a^2$   
 (c)  $y = ce^{-\frac{x}{2a}}$   
 (d)  $axy = c^2$ , where  $c$  is a constant.

13. If  $\frac{5z_1}{7z_2}$  is purely imaginary, then  $\left| \frac{2z_1 + 3z_2}{2z_1 - 3z_2} \right| =$

(a) 5/7 (b) 7/5 (c) 25/29 (d) 1

14. The mean daily profit made by a shopkeeper in a month of 30 days was ₹ 350. If the mean profit for the first twenty days was ₹ 400, then the mean profit for the last 10 days would be

(a) ₹ 200 (b) ₹ 250 (c) ₹ 800 (d) ₹ 300

15. Consider points  $A(3, 4)$  and  $B(7, 13)$ . If  $P$  be a point on the line  $y = x$  such that  $PA + PB$  is minimum, then coordinates of  $P$  are

(a)  $\left(\frac{12}{7}, \frac{12}{7}\right)$  (b)  $\left(\frac{13}{7}, \frac{13}{7}\right)$   
 (c)  $\left(\frac{31}{7}, \frac{31}{7}\right)$  (d)  $(0, 0)$

16.  $\lim_{x \rightarrow 0} \frac{\log(2+x^2) - \log(2-x^2)}{x^2} = k$ , the value of  $k$  is

(a) -1 (b) 2 (c) 1 (d) 0

17. The sum of the series  $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \dots$  is equal to

(a) 1 (b) 0 (c) -1 (d) 2

18. If  $f: R \rightarrow R, f(x+y) = f(x) \cdot f(y)$ . If  $f(1) = 1$ , then

$\sum_{k=1}^n f(k) =$   
 (a)  $n$  (b)  $n(n+1)(2n+1)/6$   
 (c) 1 (d)  $n(n+1)/2$

19. If  $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{for } x < 0 \\ c, & \text{for } x = 0 \\ \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}}, & \text{for } x > 0 \end{cases}$

is continuous at  $x = 0$ , then  $a =$

(a) 3/2 (b) -3/2 (c) 1/4 (d) -1/4

20. If  $f(x) = x^n$ , then the value of

$$\{f(1)\}^2 + \left\{\frac{f'(1)}{1!}\right\}^2 + \left\{\frac{f''(1)}{2!}\right\}^2 + \dots + \left\{\frac{f^{(n)}(1)}{n!}\right\}^2$$

is given by

(a)  ${}^{2n}C_n$  (b)  ${}^{2n}C_{n-1}$   
 (c)  ${}^{2n}C_{n+1}$  (d)  ${}^{2n}C_1$

21. For the three events  $A, B$  and  $C$ ,  $P(\text{exactly one of the events } A \text{ or } B \text{ occurs}) = P(\text{exactly one of the events } B \text{ or } C \text{ occurs}) = P(\text{exactly one of the events } C \text{ or } A \text{ occurs}) = p$  and  $P(\text{all the three events occurs simultaneously}) = p^2$ , where  $0 < p < 1/2$ . Then the probability of at least one of the three events  $A, B$  and  $C$  occurring is

(a)  $\frac{1}{2}(3p+2p^2)$  (b)  $\frac{1}{4}(p+3p^2)$   
 (c)  $\frac{1}{2}(p+3p^2)$  (d)  $\frac{1}{4}(3p+2p^2)$

22. The area of the smaller region bounded by the curves  $x^2 + y^2 = 5$  and  $y^2 = 4x$  is

(a)  $\frac{\pi}{4} + \frac{1}{6}$  (b)  $\frac{\pi}{4} - \frac{1}{6}$   
 (c)  $2\left(\frac{1}{3} - \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right)$  (d)  $2\left(\frac{1}{3} + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right)$

23. If  $a, b, c$  are real, then  $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$  is decreasing in

(a)  $\left(-\frac{2}{3}(a^2+b^2+c^2), 0\right)$   
 (b)  $\left(0, \frac{2}{3}(a^2+b^2+c^2)\right)$   
 (c)  $\left(\frac{a^2+b^2+c^2}{3}, 0\right)$  (d) None of these

24. If  $P(n): n^2 + n$  is an odd integer. It is seen that truth of  $P(n) \Rightarrow$  the truth of  $P(n+1)$ . Therefore,  $P(n)$  is true for all

(a)  $n > 1$  (b)  $n$   
 (c)  $n > 2$  (d) none of these

25. If  $y = \sqrt{\cos x^2 + \sqrt{\cos x^2 + \sqrt{\cos x^2 + \dots}}}$  to  $\infty$ ,

then  $\frac{dy}{dx}$  is

(a)  $\frac{-\sin x^2}{x(2y-1)}$  (b)  $\frac{-2x \sin x^2}{2y-1}$

- (c)  $\frac{-\sin x}{2y-1}$  (d) none of these

26. Area lying between the curves  $y^2 = 4x$  and  $y = 2x$  is

- (a)  $2/3$  (b)  $1/3$   
(c)  $1/4$  (d) none of these

27. If  $f(x)$  is a continuous function satisfying  $f(x)f(1/x) = f(x) + f(1/x)$  and  $f(1) > 0$ , then  $\lim_{x \rightarrow 1} f(x)$  is equal to

- (a) 2 (b) 1  
(c) 3 (d) none of these

28. For any integer  $n$ , the integral

$$\int_0^\pi e^{\cos^2 x} \cos^3 x (2n+1)x dx \text{ has the value}$$

- (a)  $\pi$  (b) 1  
(c) 0 (d) none of these

29. Let  $E$  and  $F$  be two independent events. The probability that both  $E$  and  $F$  happen is  $1/12$  and the probability that neither  $E$  nor  $F$  happens is  $1/2$ . Then

- (a)  $P(E) = 1/3, P(F) = 1/4$   
(b)  $P(E) = 1/2, P(F) = 1/6$   
(c)  $P(E) = 1/6, P(F) = 1/2$   
(d)  $P(E) = 1/8, P(F) = 1/3$

30. If  ${}^nC_r$  denotes the number of combinations of  $n$  things taken  $r$  at a time, then the expression

$${}^nC_0 + \sum_{k=0}^{n-1} {}^{n+k}C_{k+1} \text{ equals}$$

- (a)  ${}^{2n}C_{n-1}$  (b)  ${}^{2n}C_{n+1}$   
(c)  ${}^{2n}C_n$  (d)  ${}^nC_{n-1}$

### SOLUTIONS

1. (c): Let  $A$  be the event that the numbers of the triplets form a G.P.

Since 3 numbers can be selected from 30 numbers in  ${}^{30}C_3$  ways, therefore total number of triplets =  ${}^{30}C_3$ . Now we count the triplets (arranged in increasing order) whose terms form a G.P. by listing them as follows.

Common ratio	Triplet
2	$\{(i, 2i, 4i), 1 \leq i \leq 7\}$
3	$\{(i, 3i, 9i), 1 \leq i \leq 3\}$
4	$(1, 4, 16)$
5	$(1, 5, 25)$
3/2	$(4, 6, 9), (8, 12, 18), (12, 18, 27)$

5/2	$(4, 10, 25)$
4/3	$(9, 12, 16)$
5/3	$(9, 15, 25)$
5/4	$(16, 20, 25)$

Thus among  ${}^{30}C_3$  triplets, there are 19 triplets, whose terms form a G.P.

$$\therefore \text{Required probability } P(A) = \frac{19}{{}^{30}C_3} = \frac{19}{4060}$$

2. (c): The total number of ways of distributing  $n$  balls into  $m$  boxes is  $m^n$  which are assumed to be equally likely.

Since  $r$  balls which should go to the specified box can be chosen in  ${}^nC_r$  ways and for any such way the remaining  $(n-r)$  balls can be distributed to the  $(m-1)$  boxes in  $(m-1)^{n-r}$  ways.

$$\therefore \text{The number of favourable cases} = \binom{n}{r} (m-1)^{n-r}$$

$$\therefore P(A) = \frac{\binom{n}{r} (m-1)^{n-r}}{m^n}$$

[where  $A$  is the event that a specified box will contain  $r$  balls]

3. (c): Given that  $P(A \cup B) = 0.4$  and  $P(A \cap B) = 0.1$

$$\therefore P(A) + P(B) - P(A \cap B) = 0.4$$

$$\text{or } P(A) + P(B) = 0.4 + 0.1 = 0.5$$

$$\text{or } 1 - P(A^C) + 1 - P(B^C) = 0.5$$

$$\text{or } P(A^C) + P(B^C) = 2 - 0.5 = 1.5$$

4. (d): Let  $A$  denote the event of a successful landing of the rocket,  $B_1$  denote the event of the monitoring system indicating it correctly and  $B_2$  denote the event of its indicating unsuccessful landing.

$$\therefore P(A) = 0.4, P(B_1|A) = 0.9, P(B_2|A^C) = 0.9$$

$$\text{The required probability} = P(A|B_1) = \frac{P(B_1|A)P(A)}{P(B_1)}$$

$$\begin{aligned} \text{Now, } P(B_1) &= P(A \cap B_1) + P(A^C \cap B_1) \\ &= P(B_1|A)P(A) + P(B_1|A^C)P(A^C) \\ &= P(B_1|A)P(A) + [1 - P(B_1^C|A^C)]P(A^C) \\ &= P(B_1|A)P(A) + [1 - P(B_2|A^C)]P(A^C) \\ &= 0.9 \times 0.4 + (1 - 0.9) \times 0.6 = 0.42 \end{aligned}$$

$$\text{Hence the required probability} = \frac{0.9 \times 0.4}{0.42} = \frac{6}{7}$$

5. (a): Let  $A_i$  be the event of choosing  $i^{\text{th}}$  urn and  $B$  the event of choosing  $n$  black balls in succession and  $C$  the event of drawing  $(n+1)^{\text{th}}$  ball as black. The required probability is

$$P(C|B) = \frac{P(B \cap C)}{P(B)}$$

$$\begin{aligned} \text{Now, } P(B) &= \sum_{i=0}^N P(A_i) \cdot P(B|A_i) \\ &= \sum_{i=0}^N \frac{1}{N+1} \left(\frac{i}{n}\right)^n = \int_0^1 x^n dx \text{ when } N \rightarrow \infty \end{aligned}$$

Similarly,

$$P(B \cap C) = \sum_{i=0}^N \frac{1}{N+1} \left(\frac{i}{n}\right)^{n+1} = \int_0^1 x^{n+1} dx \text{ when } N \rightarrow \infty$$

Hence the required probability

$$\begin{aligned} &= \frac{\sum_{i=0}^N \left(\frac{i}{n}\right)^{n+1}}{\sum_{i=0}^N \left(\frac{i}{n}\right)^n} = \frac{\int_0^1 x^{n+1} dx}{\int_0^1 x^n dx} \text{ if } n \text{ is large} \\ &= \frac{n+1}{n+2} \end{aligned}$$

6. (d) : Required probability

$$= 1 - P(\text{no test is missed})$$

$$= 1 - P(\text{no test on his first day of absence and no test of his second day of absence})$$

$$= 1 - P(\text{no test of his first day of absence}) \times P(\text{no test of his second day of absence})$$

$$= 1 - \left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{21}{25}$$

7. (c) : Given that  $P(A \cap B^C) = \frac{3}{25}$

$$\text{and } P(A^C \cap B) = \frac{8}{25}, P(A) > \frac{1}{2},$$

$$\text{Let } P(A) = x, P(B) = y$$

$\therefore$  We are given that

$$P(A \cap B^C) = P(A) - P(A \cap B)$$

$$\frac{3}{25} = x - P(A \cap B)$$

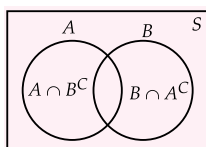
$$\frac{3}{25} = x - xy \quad [\because A \text{ and } B \text{ are independent}]$$

$$\text{Also, } P(A^C \cap B) = P(B) - P(A \cap B)$$

$$\Rightarrow \frac{8}{25} = y - xy. \text{ Hence } y = x + \frac{1}{5}$$

$$\therefore \text{ Solving } x \text{ and } y \text{ we get } x = \frac{1}{5}, y = \frac{2}{5}$$

$$\text{or } x = \frac{3}{5} \text{ and } y = \frac{4}{5}$$



As  $P(A) > \frac{1}{2}$ , we must have  $P(A) = \frac{3}{5}$  and  $P(B) = \frac{4}{5}$

$$\therefore P(A) + P(B) = \frac{7}{5}$$

8. (a) :  $P(A^C) = 0.3, P(B) = 0.4, P(AB^C) = 0.5$

$$\begin{aligned} \therefore P(A \cup B^C) &= P(A) + P(B^C) - P(A \cap B^C) \\ &= 0.7 + 0.6 - 0.5 = 0.8 \end{aligned}$$

Now,  $P(B|A \cup B^C)$

$$\begin{aligned} &= \frac{P[B \cap (A \cup B^C)]}{P(A \cup B^C)} = \frac{P[(B \cap A) \cup (B \cap B^C)]}{P(A \cup B^C)} \\ &= \frac{P(A \cap B)}{P(A \cup B^C)} = \frac{P(A) - P(A \cap B^C)}{P(A \cup B^C)} \\ &= \frac{0.7 - 0.5}{0.8} = \frac{0.2}{0.8} = \frac{1}{4} \end{aligned}$$

9. (c) : Given that  $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$

$\therefore$  Probability that only two of them can solve the problem

$$= P(A \cap B \cap C^C) + P(A \cap B^C \cap C) + P(A^C \cap B \cap C)$$

$$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{6}{24} = \frac{1}{4}$$

10. (a) :  $P(E_1) = P(E_2) = \frac{1}{2}$

$$\text{and } P(R|E_1) = \frac{3}{5}, P(R|E_2) = \frac{5}{9}$$

By Bayes' theorem

$$\begin{aligned} P(E_2|R) &= \frac{P(E_2) \cdot P(R|E_2)}{P(E_1) \cdot P(R|E_1) + P(E_2) \cdot P(R|E_2)} \\ &= \frac{\frac{1}{2} \cdot \frac{5}{9}}{\frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{5}{9}} = \frac{\frac{5}{9}}{\frac{3}{5} + \frac{5}{9}} = \frac{25}{27+25} \end{aligned}$$

$$P(E_2|R) = \frac{25}{52}$$

11. (a) : Determinant of coefficients

$$\begin{vmatrix} \lambda & -1 & \cos \theta \\ 3 & 1 & 2 \\ \cos \theta & 1 & 2 \end{vmatrix} = \cos \theta - \cos^2 \theta + 6$$

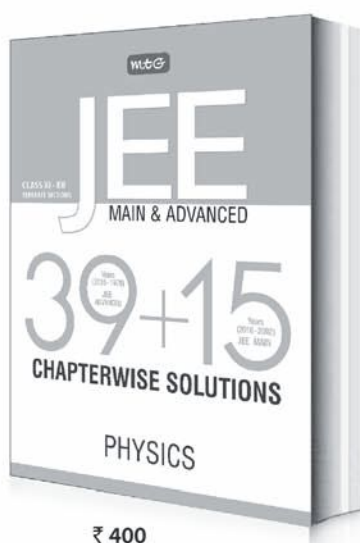
and this is positive for all  $\theta$  since  $|\cos \theta| \leq 1$ . The only solution is therefore the trivial solution.

12. (c) :  $2y \cdot \frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$

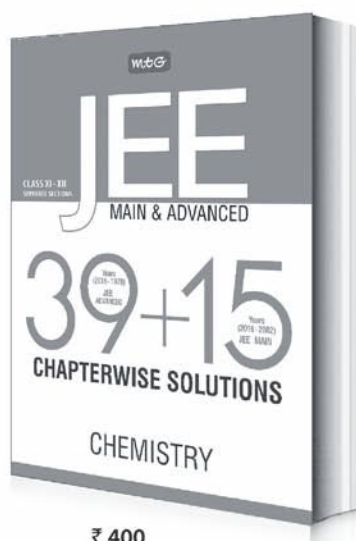
For orthogonal trajectory,



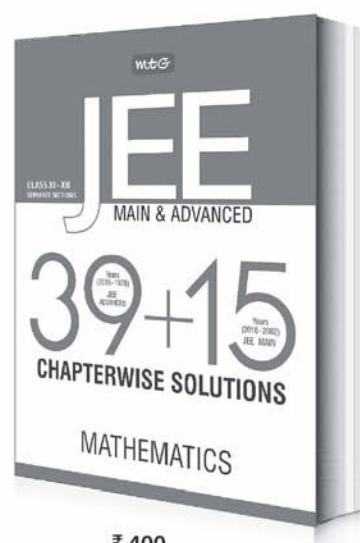
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$$-\frac{dx}{dy} = \frac{2a}{y} \Rightarrow \int \frac{dy}{y} = \int -\frac{1}{2a} dx$$

$$\Rightarrow \ln y = -\frac{x}{2a} + k \Rightarrow y = ce^{-\frac{x}{2a}}$$

13. (d) : Let  $\frac{5z_1}{7z_2} = Ki$  ( $K \in R$ ), then  $\frac{z_1}{z_2} = \left(\frac{7K}{5}\right)i$

Consider  $\left| \frac{2z_1 + 3z_2}{2z_1 - 3z_2} \right| = \left| \frac{2\left(\frac{z_1}{z_2}\right) + 3}{2\left(\frac{z_1}{z_2}\right) - 3} \right| = \left| \frac{\frac{14K}{5}i + 3}{\frac{14K}{5}i - 3} \right| = 1$

14. (b) :  $30 \times 350 = 20 \times 400 + 10 \times x$   
 $\Rightarrow 10x = 10500 - 8000 \Rightarrow 10x = 2500 \Rightarrow x = 250$

15. (c) : Consider a point  $A'$ , the image of  $A$  through  $y = x$

$\therefore$  Coordinates of  $A' = (4, 3)$   
 [Notice that  $A$  and  $B$  lie to the same side with respect to  $y = x$ ].

Then  $PA = PA'$

Thus,  $PA + PB$  is minimum, if  $PA' + PB$  is minimum, if  $P, A', B$  are collinear.

Now,  $AB$  is  $y - 3 = \frac{13-3}{7-4}(x-4) \Rightarrow 3y - 10x + 31 = 0$

It intersects  $y = x$  at  $\left(\frac{31}{7}, \frac{31}{7}\right)$ , which is the required point  $P$ .

16. (c) :  $\lim_{x \rightarrow 0} \frac{1}{x^2} [\log(2+x^2) - \log(2-x^2)]$

$$= \lim_{x \rightarrow 0} \frac{1}{x^2} \left[ \log\left(\frac{2+x^2}{2}\right) - \log\left(\frac{2-x^2}{2}\right) \right]$$

$$= \lim_{x \rightarrow 0} \left[ \log\left(1 + \frac{x^2}{2}\right)^{1/x^2} - \log\left(1 - \frac{x^2}{2}\right)^{1/x^2} \right]$$

$$= \log e^{1/2} - \log e^{-1/2} = \frac{1}{2} + \frac{1}{2} = 1$$

17. (a) : The given series is,

$$\sum_{r=1}^{\infty} \frac{(2r+1)}{r^2(r+1)^2} = \sum_{r=1}^{\infty} \frac{1}{r^2} - \sum_{r=1}^{\infty} \frac{1}{(r+1)^2} = 1$$

18. (a) :  $f(1) = 1 \Rightarrow f(2) = 1 \Rightarrow f(3) = 1$   
 $f(r) = 1 \quad \forall \quad r = 1, 2, \dots, n$

$$\sum_{k=1}^n f(k) = n$$

19. (b) :  $f(0-0) = \lim_{h \rightarrow 0} f(0-h)$

$$= \lim_{h \rightarrow 0} \frac{\sin(a+1)(-h) + \sin(-h)}{0-h} = \lim_{h \rightarrow 0} \frac{(a+1+1)(-h)}{(-h)} = a+2$$

$$f(0+0) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{(h+bh^2)^{1/2} - h^{1/2}}{bh^{3/2}}$$

$$= \lim_{h \rightarrow 0} \frac{b^{1/2} \left[ 1 + \frac{bh}{2} - 1 \right]}{bh \cdot h^{1/2}} = \frac{1}{2}$$

$\therefore$  The given function is continuous  $f(0-0) = f(0+0)$ .

$$\therefore a+2 = \frac{1}{2}, \quad a = -\frac{3}{2}$$

20. (a) :  $f(x) = x^n \Rightarrow \frac{f^r(1)}{r!} = {}^nC_r$

$$\sum_{r=0}^n ({}^nC_r)^2 = {}^{2n}C_n$$

21. (a) :  $P(A \cup B \cup C) = \sum P(A) - P(AB) - P(BC) - P(AC) + P(A \cap B \cap C)$

$$= \frac{1}{2} \left\{ \begin{aligned} &P(A) - P(AB) + P(B) - P(BC) + P(C) \\ &- P(AB) + P(C) - P(BC) + P(A) - P(AC) \\ &+ P(C) - P(AC) \end{aligned} \right\} + P(ABC)$$

$$= \frac{1}{2} \{ P(A\bar{B}) + P(\bar{A}B) + P(A\bar{C}) + P(\bar{A}C) + P(\bar{B}C) + P(\bar{C}B) \} + P(ABC)$$

$$= \frac{1}{2} (3p) + p^2 = \frac{1}{2} (3p + 2p^2)$$

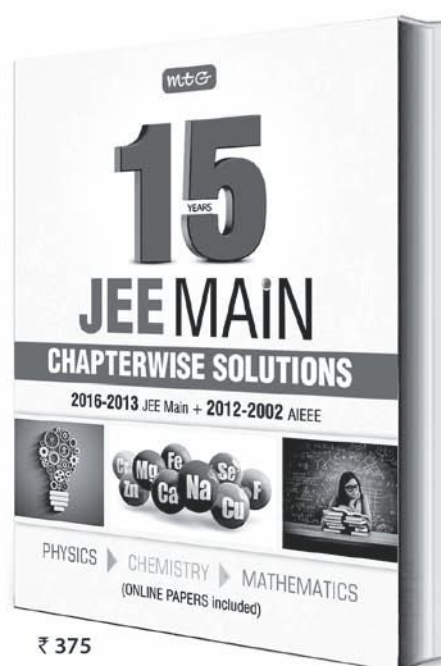
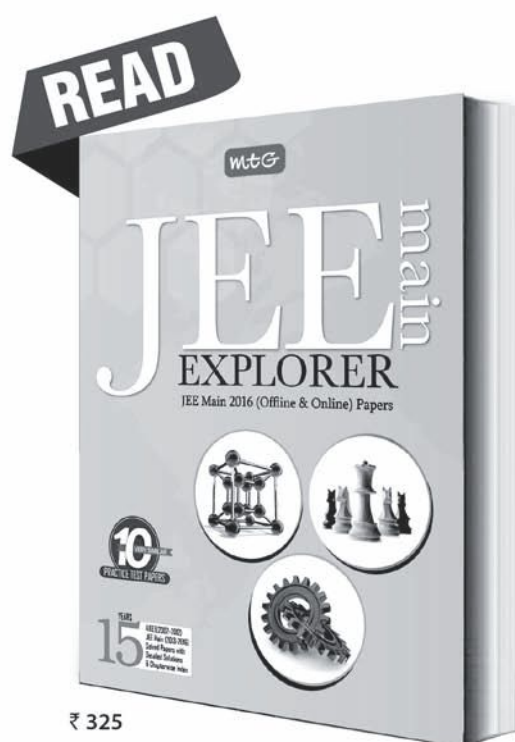
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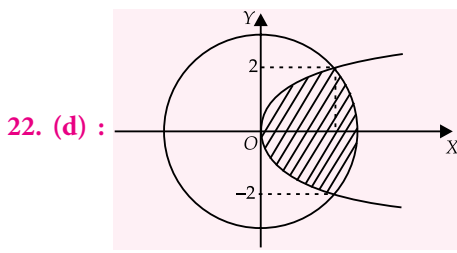
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$$\begin{aligned} \text{Area} &= \int_{-2}^2 \left( \sqrt{5-y^2} - \frac{y^2}{4} \right) dy \\ &= 2 \int_0^2 \left( \sqrt{5-y^2} - \frac{y^2}{4} \right) dy = 2 \left( \frac{1}{3} + \frac{5}{2} \sin^{-1} \left( \frac{2}{\sqrt{5}} \right) \right) \end{aligned}$$

23. (a) :

$$\begin{aligned} f'(x) &= \begin{vmatrix} 1 & 0 & 0 \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix} + \begin{vmatrix} x+a^2 & ab & ac \\ 0 & 1 & 0 \\ ac & bc & x+c^2 \end{vmatrix} \\ &\quad + \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ 0 & 0 & 1 \end{vmatrix} \\ &= (x+b^2)(x+c^2) - b^2c^2 + (x+a^2)(x+c^2) - a^2c^2 \\ &\quad + (x+a^2)(x+b^2) - a^2b^2 \\ &= 3x^2 + 2x(a^2 + b^2 + c^2) \end{aligned}$$

$f(x)$  will be decreasing when  $f'(x) < 0$

$$\Rightarrow 3x^2 + 2x(a^2 + b^2 + c^2) < 0$$

$$\Rightarrow x \in \left( -\frac{2}{3}(a^2 + b^2 + c^2), 0 \right)$$

24. (d) : The sufficient condition for the statement :  $P(n)$  to be true for  $n \geq a$  is

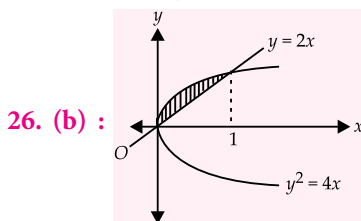
(i) truth of  $P(n) \Rightarrow$  truth of  $P(n+1)$

(ii)  $P(a)$  is true. Hence answer is none of these.

25. (b) :  $y = \sqrt{\cos x^2 + y} \Rightarrow y^2 - y = \cos x^2$

$$\therefore 2y \frac{dy}{dx} - \frac{dy}{dx} = -\sin x^2 \cdot 2x$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2x \sin x^2}{2y-1}$$



$$\begin{aligned} (2x)^2 &= 4x \Rightarrow 4x^2 = 4x \\ \Rightarrow 4x^2 - 4x &= 0 \Rightarrow 4x(x-1) = 0 \Rightarrow x = 0, 1 \end{aligned}$$

$$\text{Required area} = \int_0^1 (2\sqrt{x} - 2x) dx$$

$$= 2 \cdot \frac{2}{3} x^{3/2} - x^2 \Big|_0^1 = \frac{4}{3} - 1 = \frac{1}{3}$$

27. (a) : Since  $f$  is continuous function so,

$$\lim_{x \rightarrow 1} f(x) = f(1).$$

Put  $x = 1$  in the given equation we have

$$(f(1))^2 = 2f(1), \text{ so } f(1) = 0 \text{ or } 2.$$

Since  $f(1) > 0$ , so  $f(1) = 2$ .

28. (c)

29. (a) : Let  $P(E) = x, P(F) = y, xy = \frac{1}{12}$

$$(1-x)(1-y) = \frac{1}{2}, \text{ or } 1 - (x+y) + xy = \frac{1}{2}$$

$$x+y = \frac{1}{2} + \frac{1}{12} = \frac{7}{12}; x+y = \frac{7}{12}, xy = \frac{1}{12}$$

$$P(E) = \frac{1}{3} \text{ and } P(F) = \frac{1}{4}$$

$$\text{or, } P(E) = \frac{1}{4} \text{ and } P(F) = \frac{1}{3}$$

30. (c) :  ${}^nC_0 + \sum_{k=0}^{n-1} {}^{n+k}C_{k+1} = ({}^nC_0 + {}^nC_1) + {}^{n+1}C_2$

$$+ {}^{n+2}C_3 + \dots + {}^{n+n-1}C_n$$

$$= ({}^{n+1}C_1 + {}^{n+1}C_2) + {}^{n+2}C_3 + \dots + {}^{2n-1}C_n$$

$$= ({}^{n+2}C_2 + {}^{n+2}C_3) + \dots + {}^{2n-1}C_n$$

$$\text{Similarly, } {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n$$

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JEE Advanced	21 <sup>st</sup> May

# PRACTICE PAPER

# JEE ADVANCED

## SECTION-1

### SINGLE OPTION CORRECT TYPE

- Consider the two statements.  
 $S_1$  : The number of solutions to the equation  $e^x = x^2$  is 1.  
 $S_2$  : The number of solutions to the equation  $e^x = x^3$  is 1.  
 (a)  $S_1$  is true,  $S_2$  is false.  
 (b)  $S_1$  is true,  $S_2$  is true.  
 (c)  $S_1$  is false,  $S_2$  is true.  
 (d)  $S_1$  is false,  $S_2$  is also false.
- The probability that a random chosen divisor of  $30^{39}$  is a multiple of  $30^{29}$  is  
 (a)  $\left(\frac{11}{40}\right)^3$  (b)  $\left(\frac{21}{40}\right)^3$   
 (c)  $\left(\frac{11}{40}\right)^5$  (d)  $\left(\frac{21}{40}\right)^5$
- 4 persons are playing a game of toy-gun shooting. At exactly midnight, each person randomly chooses one of the other three and shoots him. The probability that exactly 2 persons are shot is  
 (a)  $\frac{8}{27}$  (b)  $\frac{4}{9}$  (c)  $\frac{2}{3}$  (d)  $\frac{1}{4}$
- In a certain lottery, 7 balls are drawn at random from  $n$  balls numbered 1 through  $n$ . If the probability that no pair of consecutive numbers is drawn equals the probability of drawing exactly one pair of consecutive numbers, then  $n =$   
 (a) 50 (b) 60 (c) 54 (d) 64
- If  $B$  is an idempotent matrix satisfying the condition  $(I - aB)^{-1} = I - 3B$ , where  $I$  is unit matrix of same order as  $B$  and  $a \in R$  then  $2a =$   
 (a) 1 (b) 2 (c) -1 (d) 3
- Let  $A, B, C, D$  be distinct points on a circle with centre  $O$ . If there exists non-zero real numbers  $x$  and  $y$  such that  $|x\overrightarrow{OA} + y\overrightarrow{OB}| = |x\overrightarrow{OB} + y\overrightarrow{OC}| = |x\overrightarrow{OC} + y\overrightarrow{OD}| = |x\overrightarrow{OD} + y\overrightarrow{OA}|$ , then  $ABCD$  is  
 (a) a square (b) a trapezium  
 (c) a rectangle (d) none of these
- An ellipse has foci at  $(9, 20)$  and  $(49, 55)$  in the  $xy$  plane and is tangent to  $x$ -axis. The length of its major axis is  
 (a) 70 (b) 75 (c) 80 (d) 85
- A rectangular box has faces parallel to co-ordinate planes. If two of its vertices are  $(1, -1, 0)$  and  $(2, 3, 6)$  then volume of the box is  
 (a) 20 (b) 22 (c) 24 (d) 26
- The number of distinct real number pairs  $(a, b)$  such that  $a + b \in \text{integers}$  and  $a^2 + b^2 = 2$  is/are  
 (a) 10 (b) 6 (c) 2 (d) 4
- Let  $n \geq 3$  be an integer and  $z = \text{cis}\left(\frac{2\pi}{n}\right)$ . Consider the sets  $A = \{1, z, z^2, \dots, z^{n-1}\}$  and  $B = \{1, 1 + z, 1 + z + z^2, \dots, 1 + z + z^2 + \dots + z^{n-1}\}$ . The number of elements in the set  $(A \cap B)$  when  $n$  is even is  
 (a) 0 (b) 1 (c) 2 (d) 3
- Let  $a$  and  $b$  be positive real numbers such that  $a[a] = 17$  and  $b[b] = 11$  then  $(a - b) =$   
 (a)  $3/7$  (b)  $4/7$  (c)  $7/12$  (d)  $5/12$   
 ( $[\cdot]$  denotes greatest integer function)
- Let a function  $f(x)$ ,  $x \neq 0$  be such that  $f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right)$  then  $f(x)$  can be  
 (a)  $1 - x^4 + x^3$  (b)  $\sqrt{|x| - x + 1}$   
 (c)  $\frac{\pi}{2 \tan^{-1}|x|}$  (d)  $\frac{2x}{1 + \lambda \log|x|}$

13. Assume that  $a (> 1)$  is a root of the equation  $x^3 - x - 1 = 0$  then  $\sqrt[3]{3a^2 - 4a} + \sqrt[3]{3a^2 + 4a + 2} =$   
 (a) 1 (b) 2 (c)  $2a$  (d)  $a^{1/3}$

14. The maximum and minimum of the function  $6\sin x \cos y + 2\sin x \sin y + 3\cos x$  is  
 (a) 7 and -7 (b) 5 and -5  
 (c)  $6 + \sqrt{13}$  and  $6 - \sqrt{13}$   
 (d)  $5 + \sqrt{2}$  and  $5 - \sqrt{2}$

15. For a real number  $\alpha$ , let

$$I(\alpha) = \int_0^{\pi} \log(1 - 2\alpha \cos x + \alpha^2) dx \text{ then } I(\alpha) =$$

(a)  $I(\alpha^2)$  (b)  $\frac{1}{2}I(\alpha^2)$   
 (c)  $2I(\alpha^2)$  (d)  $I(-\alpha^2)$

16. The minimum of the expression

$$\left(x^2 + \frac{16}{x^2}\right) - 2\left[(1 + \cos\theta)x + \frac{4}{x}(1 + \sin\theta)\right] + (3 + 2\cos\theta + 2\sin\theta)$$

for  $x > 0$  and  $\theta \in [0, 2\pi]$  is

- (a)  $2\sqrt{2} - 1$  (b)  $2\sqrt{2} + 3$   
 (c)  $2\sqrt{2} + 1$  (d) none of these
17. The area of the region contained by all the points  $(x, y)$  such that  $x^2 + y^2 \leq 100$  and  $\sin(x + y) \geq 0$  is  
 (a)  $10\pi$  (b)  $25\pi$  (c)  $50\pi$  (d)  $100\pi$

## SECTION-2

### COMPREHENSION TYPE

#### Passage-1

Let  $a$  be a fixed real number,  $a \in (0, \pi)$  and

$$I_r = \int_{-a}^a \frac{1 - r \cos u}{1 - 2r \cos u + r^2} du$$

18.  $I_1 =$   
 (a) 0 (b)  $a$  (c)  $2a$  (d)  $-a$
19.  $\lim_{r \rightarrow 1^+} I_r =$   
 (a)  $a - \pi$  (b)  $a + \pi$  (c)  $a$  (d)  $-a$
20.  $\lim_{r \rightarrow 1^-} I_r =$   
 (a)  $a - \pi$  (b)  $a + \pi$  (c)  $a$  (d)  $-a$

#### Passage-2

Let  $r_1 < 0 < r_2 < r_3$  be the real roots of  $8x^3 - 6x + \sqrt{3} = 0$  and let  $s_1 < 0 < s_2 < s_3$  be the real roots of  $8x^3 - 6x + 1 = 0$ , then

21.  $4r_2^2 - 4r_2^4 =$   
 (a)  $r_1^2$  (b)  $r_2^2$  (c)  $r_3^2$  (d)  $r_3^2 - r_1^2$
22.  $r_1^2 + s_2^2 =$   
 (a) 1 (b) 2 (c) 4 (d) 8

## SOLUTIONS

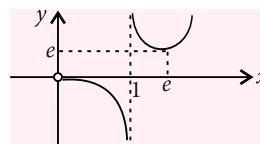
1. (a) : For  $x < 0$ ,  $e^x = x^2$  has one solution and  $e^x = x^3$  has no solution.

For  $x > 0$ , consider the function  $f(x) = \frac{x}{\log x}$

$$\therefore f'(x) = \frac{\log x - 1}{(\log x)^2}$$

So, for  $x > e$ ,  $f(x)$  increases and for  $x < e$ ,  $f(x)$  decreases and moreover  $x = 1$  is an asymptote.

So, graph of  $f(x)$  is



Now,  $y = 2$  does not intersect the graph but  $y = 3$  meets the graph at 2 different points.

Hence, in total  $e^x = x^2$  intersects once and  $e^x = x^3$  intersects twice.

2. (a) :  $30^{39} = 2^{39} \times 3^{39} \times 5^{39}$ . So,  $40 \times 40 \times 40$  divisors in total.

$2^a 3^b 5^c$  to be a multiple of  $30^{29}$ , we must have  $a \in [29, 39]$ ,  $b \in [29, 39]$ ,  $c \in [29, 39]$  i.e. 11 choices for each  $a, b, c$ .

$$\text{Hence, required probability} = \frac{11 \times 11 \times 11}{40 \times 40 \times 40} = \left(\frac{11}{40}\right)^3$$

3. (a) : Let the 4 persons were  $A, B, C, D$ . First, we choose which two were shot  $= {}^4C_2$ . Let  $C, D$  were shot. Then  $C \rightleftharpoons D$  and  $A \rightarrow C$  or  $D$  and  $B \rightarrow C$  or  $D$ . Any person shooting any other person has probability  $= 1/3$ . So, probability that  $C, D$  were shot

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{81}$$

[for  $C, D$ ] [for  $A, B$ ]

Hence, probability that exactly 2 persons were shot

$$= {}^4C_2 \times \frac{4}{81} = \frac{8}{27}$$

4. (c) : There are  ${}^{n-6}C_7$  ways to draw 7 balls so that no two balls are consecutive and  $(n-6) \times {}^{n-7}C_5$  ways to draw 7 balls so that there is exactly one pair of consecutive balls. Hence,  ${}^{n-6}C_7 = (n-6) \times {}^{n-7}C_5$  gives  $n = 54$

5. (d): From given relation,  $I = (I - 3B)(I - aB)$   
*i.e.*,  $I = I - aB - 3B + 3aB = I + 2aB - 3B$   
Hence,  $2a = 3$

6. (a): Squaring,  $|x\overline{OA} + y\overline{OB}|^2 = |x\overline{OB} + y\overline{OC}|^2$   
 $\Rightarrow x^2\overline{OA} \cdot \overline{OA} + y^2\overline{OB} \cdot \overline{OB} + 2xy\overline{OA} \cdot \overline{OB}$   
 $= x^2\overline{OB} \cdot \overline{OB} + y^2\overline{OC} \cdot \overline{OC} + 2xy\overline{OB} \cdot \overline{OC}$   
*i.e.*,  $x^2r^2 + y^2r^2 + 2xy\overline{OA} \cdot \overline{OB}$   
 $= x^2r^2 + y^2r^2 + 2xy\overline{OB} \cdot \overline{OC}$

Hence, ABCD is a square.

7. (d): Let  $F_1$  and  $F_2$  be the foci and  $P$  be the point of tangency. Let  $F_1'$  be the image of  $F_1$  in  $x$ -axis then

$$F_1P + F_2P = F_1'P + F_2P = F_1'F_2 = 2a$$

$$= \sqrt{(40)^2 + (75)^2} = 85$$

8. (c): Vertices are  $(1, -1, 0)$  and  $(2, 3, 6)$ . Notice that these must be diagonal vertices.

Making  $(1, -1, 0)$  as origin, we have  $(2, 3, 6)$  as  $(1, 4, 6)$ .

Hence, dimensions of the box are  $6 \times 4 \times 1$

*i.e.* Volume = 24 cubic units

9. (b): Using, R.M.S  $\geq$  A.M., we have  $|a + b| \leq 2$   
So,  $a + b \in \{-2, -1, 0, 1, 2\}$  and  $a^2 + b^2 = 2$  gives

$$(a, b) = (1, -1), (-1, 1), \left(\frac{1 \pm \sqrt{3}}{2}, \frac{1 \mp \sqrt{3}}{2}\right),$$

$$\left(\frac{-1 \pm \sqrt{3}}{2}, \frac{-1 \mp \sqrt{3}}{2}\right)$$

10. (c): Clearly,  $1 \in A \cap B$ . Let  $w \in A \cap B$ ,  $w \neq 1$   
As an element of  $B$ , for some  $k = 1, 2, 3, \dots (n-1)$

$$w = 1 + z + z^2 + \dots + z^k = \frac{1 - z^{k+1}}{1 - z}$$

For  $w \in A$ , we have  $|w| = 1$

$$\Rightarrow \left| \frac{1 - z^{k+1}}{1 - z} \right| = 1 \Rightarrow \sin \frac{(k+1)\pi}{n} = \sin \left( \frac{\pi}{n} \right)$$

*i.e.*  $k = n - 2$

$$\text{So, } w = \frac{1 - 1/z}{1 - z} = \frac{-1}{z}$$

$$\text{So, } A \cap B = \left\{ 1, \frac{-1}{z} \right\}$$

11. (c):  $a[a] = 17 \Rightarrow a \in (4, 5)$ . So,  $[a] = 4$

and  $a = \frac{17}{4}$ . Similarly,  $b[b] = 11 \Rightarrow b = \frac{11}{3}$

Hence,  $a - b = \frac{7}{12}$

12. (2): Rearranging the given fractional equation, we have

$$f\left(\frac{1}{x}\right) - 1 = \frac{1}{f(x) - 1},$$

which is satisfied by only  $f(x) = \frac{\pi}{2 \tan^{-1} x}$ , out of the four options.

13. (b): Since,  $a^3 - a - 1 = 0$ , we have

$$\sqrt[3]{3a^2 - 4a} = \sqrt[3]{3a^2 - 4a - (a^3 - a - 1)} = \sqrt[3]{(1-a)^3} = 1 - a$$

$$\text{and } \sqrt[3]{3a^2 + 4a + 2 + a^3 - a - 1} = \sqrt[3]{(1+a)^3} = 1 + a$$

Hence, given expression = 2

14. (a): Using Cauchy-Schwarz inequality

$$\therefore (x_1y_1 + x_2y_2 + x_3y_3)^2 \leq (x_1^2 + x_2^2 + x_3^2)(y_1^2 + y_2^2 + y_3^2)$$

$$\text{We have, } (6\sin x \cos y + 2\sin x \sin y + 3\cos x)^2$$

$$\leq (6^2 + 2^2 + 3^2)((\sin x \cos y)^2 + (\sin x \sin y)^2 + \cos^2 x)$$

$$\text{i.e. } (6\sin x \cos y + 2\sin x \sin y + 3\cos x)^2 \leq 49$$

Hence, maximum = 7 and minimum = -7

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**15. (b) :** We have,  $I(\alpha) = I(-\alpha)$  by putting  $x = \pi - y$ .  
Now, we have

$$(1 - 2\alpha \cos x + \alpha^2)(1 + 2\alpha \cos x + \alpha^2) = 1 - 2\alpha^2 \cos^2 x + \alpha^4$$

$$\text{So, } I(\alpha) + I(-\alpha) = \int_0^\pi \log(1 - 2\alpha^2 \cos 2x + \alpha^4) dx$$

$$= \frac{1}{2} \int_0^{2\pi} \log(1 - 2\alpha^2 \cos y + \alpha^4) dy, \quad (y = 2x)$$

$$= \frac{1}{2} I(\alpha^2) + \frac{1}{2} \int_\pi^{2\pi} \log(1 - 2\alpha^2 \cos y + \alpha^4) dy$$

Putting  $y = 2\pi - t$  we have

$$\int_\pi^{2\pi} \log(1 - 2\alpha^2 \cos y + \alpha^4) dy = I(\alpha^2)$$

$$\text{Hence, } I(\alpha) + I(-\alpha) = \frac{1}{2} I(\alpha^2) + \frac{1}{2} I(\alpha^2)$$

$$\text{So, } I(\alpha) = I(-\alpha) = \frac{1}{2} I(\alpha^2)$$

**16. (d) :** The given expression can be rearranged as

$$[x - (1 + \cos \theta)]^2 + \left[ \frac{4}{x} - (1 + \sin \theta) \right]^2$$

which is square of distance between point  $\left(x, \frac{4}{x}\right)$  on hyperbola  $xy = 4$  and point  $(1 + \cos \theta, 1 + \sin \theta)$  on a circle centre  $(1, 1)$  and radius 1. Clearly the minimum distance occurs at  $(2, 2)$  on the hyperbola and  $\left(1 + \frac{1}{\sqrt{2}}, 1 + \frac{1}{\sqrt{2}}\right)$  on the circle.

$$\text{So, minimum distance} = 2\sqrt{\left(2 - \left(1 + \frac{1}{\sqrt{2}}\right)\right)^2} = \sqrt{2} - 1$$

**17. (c) :** Using the symmetry, required area of the region is  $\frac{1}{2} \times 100\pi = 50\pi$  sq. units

$$\text{18. (b) : } I_1 = \int_{-a}^a \frac{1 - \cos u}{2 - 2\cos u} du = \int_{-a}^a \frac{1}{2} du = a$$

$$\text{19. (a) : For } r > 0, I_r = a + \frac{1}{2}(1 - r^2) \cdot \int_{-a}^a \frac{du}{1 - 2r \cos u + r^2}$$

$$I_r = a + \frac{(1 - r^2)}{2} \cdot J_r \quad (\text{let})$$

$$J_r = \int_{-a}^a \frac{du}{1 - 2r \cos u + r^2}$$

Putting  $\tan(u/2) = t$  and simplifying

$$J_r = \frac{4}{|1 - r^2|} \tan^{-1} \left( \left| \frac{1+r}{1-r} \right| \tan \left( \frac{a}{2} \right) \right)$$

$$\text{So, } I_r = a + \frac{(1 - r^2)}{2} \cdot \frac{4}{|1 - r^2|} \tan^{-1} \left( \left| \frac{1+r}{1-r} \right| \tan \frac{a}{2} \right)$$

$$\begin{aligned} \text{So, } \lim_{r \rightarrow 1^+} I_r &= a - 2 \tan^{-1} \left( \frac{1+r}{r-1} \tan \frac{a}{2} \right)_{r \rightarrow 1} \\ &= a - 2 \cdot \frac{\pi}{2} = a - \pi \end{aligned}$$

$$\begin{aligned} \text{20. (b) : } \lim_{r \rightarrow 1^-} I_r &= a + 2 \tan^{-1} \left( \frac{1+r}{1-r} \tan \frac{a}{2} \right)_{r \rightarrow 1} \\ &= a + 2 \cdot \frac{\pi}{2} = a + \pi \end{aligned}$$

**21. (c) :** Putting  $x = \sin \theta$  in  $8x^3 - 6x + \sqrt{3} = 0$ , we have

$$\sin 3\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9} \quad \text{in } [0, 2\pi]$$

$$\text{Hence, } r_1 = \sin \frac{13\pi}{9} = -\sin \frac{4\pi}{9}, r_2 = \sin \frac{\pi}{9}, r_3 = \sin \frac{2\pi}{9}$$

Similarly, putting  $x = \cos \theta$  in  $8x^3 - 6x + 1 = 0$ ,

$$\text{we have, } \cos 3\theta = -\frac{1}{2}$$

$$\text{So, } s_1 = \cos \frac{8\pi}{9} = -\cos \frac{\pi}{9}, \text{ and}$$

$$s_2 = \cos \frac{4\pi}{9}, s_3 = \cos \frac{2\pi}{9}$$

$$\text{Hence, } 4r_2^2 - 4r_2^4 = 4 \sin^2 \frac{\pi}{9} - 4 \sin^4 \frac{\pi}{9} = \sin^2 \frac{2\pi}{9} = r_3^2$$

$$\text{22. (a) : } r_1^2 + s_2^2 = \sin^2 \frac{4\pi}{9} + \cos^2 \frac{4\pi}{9} = 1$$

## MPP-8 CLASS XII

## ANSWER KEY

1. (c) 2. (d) 3. (d) 4. (a) 5. (b)  
6. (c) 7. (a,b,c) 8. (a,b,c) 9. (c,d) 10. (a,b)  
11. (a,b,c,d) 12. (a,b) 13. (b,c) 14. (a)  
15. (d) 16. (d) 17. (2) 18. (1) 19. (1)  
20. (3)

# YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

**1.** The base of a  $\Delta$  is divided into three equal parts. If  $t_1, t_2, t_3$  be the tangents of the angles subtended by these parts at the opposite vertex, prove that:

$$\left(\frac{1}{t_1} + \frac{1}{t_2}\right)\left(\frac{1}{t_2} + \frac{1}{t_3}\right) = 4\left(1 + \frac{1}{t_2^2}\right) \quad (\text{Yogesh, Delhi})$$

**Ans.** Let the points  $P$  and  $Q$  divide the side  $BC$  in three equal parts such that  $BP = PQ = QC = x$

Also let,  $\angle BAP = \alpha$ ,  $\angle PAQ = \beta$ ,  $\angle QAC = \gamma$  and  $\angle AQC = \theta$

From question,

$$\tan \alpha = t_1, \tan \beta = t_2, \tan \gamma = t_3.$$

Applying,  $m : n$  rule in triangle  $ABC$  we get,

$$(2x + x) \cot \theta = 2x \cot (\alpha + \beta) - x \cot \gamma \quad \dots(1)$$

From  $\Delta APC$ , we get

$$(x + x) \cot \theta = x \cot \beta - x \cot \gamma \quad \dots(2)$$

Dividing (1) by (2), we get

$$\frac{3}{2} = \frac{2 \cot (\alpha + \beta) - \cot \gamma}{\cot \beta - \cot \gamma}$$

$$\text{or } 3 \cot \beta - \cot \gamma = \frac{4(\cot \alpha \cdot \cot \beta - 1)}{\cot \beta + \cot \alpha}$$

$$\text{or } 4 + 4 \cot^2 \beta = \cot^2 \beta + \cot \alpha \cdot \cot \beta + \cot \beta \cdot \cot \gamma + \cot \gamma \cdot \cot \alpha$$

$$\text{or } 4(1 + \cot^2 \beta) = (\cot \beta + \cot \alpha)(\cot \beta + \cot \gamma)$$

$$\text{or } 4\left(1 + \frac{1}{t_2^2}\right) = \left(\frac{1}{t_1} + \frac{1}{t_2}\right)\left(\frac{1}{t_2} + \frac{1}{t_3}\right)$$

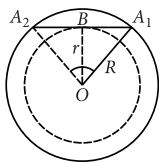
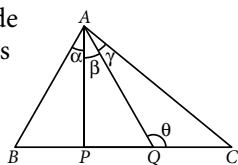
**2.** If a regular polygon of  $n$  sides has the circumradius  $R$  and inradius  $r$  then prove that each side of the polygon is equal to

$$2(R + r) \tan \frac{\pi}{2n}. \quad (\text{Raman, Gujarat})$$

**Ans.** Let  $A_1A_2$  be a side and  $O$  be the centre. Let  $OB \perp A_1A_2$ .

Clearly,  $OA_1 = R$  and  $OB = r$ .

$$\text{Also, } \angle A_1OA_2 = \frac{2\pi}{n} \text{ and } \angle A_1OB = \frac{\pi}{n}.$$



$$\text{In } \Delta A_1OB, \frac{r}{R} = \cos \angle A_1OB = \cos \frac{\pi}{n} \quad \dots(1)$$

$$\text{and } \frac{A_1B}{OB} = \tan \frac{\pi}{n}, \text{ i.e., } A_1B = r \tan \frac{\pi}{n}$$

$$\therefore A_1A_2 = 2A_1B = 2r \tan \frac{\pi}{n} \quad \dots(2)$$

$$\text{Now, } 2(R + r) \tan \frac{\pi}{2n} = 2 \left( \frac{r}{\cos \frac{\pi}{n}} + r \right) \tan \frac{\pi}{2n}, \text{ (from (1))}$$

$$= 2r \left( \frac{1 + \cos \frac{\pi}{n}}{\cos \frac{\pi}{n}} \right) \cdot \frac{1 - \cos \frac{\pi}{n}}{\sin \frac{\pi}{n}} \left\{ \because \tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} \right\}$$

$$= 2r \cdot \frac{1 - \cos^2 \frac{\pi}{n}}{\cos \frac{\pi}{n} \cdot \sin \frac{\pi}{n}} = 2r \cdot \frac{\sin^2 \frac{\pi}{n}}{\cos \frac{\pi}{n} \cdot \sin \frac{\pi}{n}}$$

$$= 2r \tan \frac{\pi}{n} = A_1A_2, \text{ (from (2)).}$$

**3.** In how many ways can two distinct subsets of the set  $A$  of  $k$  ( $k \geq 3$ ) elements be selected so that they have exactly two common elements? **(Akhil, Assam)**

**Ans.** Let the two subsets be called  $P$  and  $Q$ . The elements for the two sets will be selected as follows:

(i) 2 elements out of  $k$  elements for both the sets.

This can be done in  ${}^kC_2$  ways.

(ii)  $r$  elements for the subset  $P$  from  $k - 2$  elements and any number of elements for  $Q$  from the remaining  $k - 2 - r$  elements. Here  $r$  can vary from 0 to  $k - 2$ .

For a fixed  $r$ , the number of selections

$$= {}^{k-2}C_r \cdot 2^{k-2-r}, \text{ (because the number of selections of any number of things from } n \text{ things is } 2^n.)$$

If  $r$  varies from 0 to  $k - 2$ , the total number of selections

$$= \sum_{r=0}^{k-2} {}^{k-2}C_r \cdot 2^{k-2-r} - 1,$$

excluding the case when both the subsets are equal having only the two common elements.

But every pair of  $P, Q$  is appearing twice like  $\{a_1, a_2, a_3\}, \{a_1, a_2, a_4, a_5\}$  and  $\{a_1, a_2, a_4, a_5\}, \{a_1, a_2, a_3\}$ .

Hence, the required number of ways

$$= {}^kC_2 \times \frac{1}{2} \left( \sum_{r=0}^{k-2} {}^{k-2}C_r \cdot 2^{k-2-r} - 1 \right)$$

$$= \frac{1}{2} \cdot \frac{k(k-1)}{2} \cdot [({}^{k-2}C_0 \cdot 2^{k-2} + {}^{k-2}C_1 \cdot 2^{k-3} + {}^{k-2}C_2 \cdot 2^{k-4} + \dots + {}^{k-2}C_{k-2}) - 1]$$

$$= \frac{k(k-1)}{4} \cdot \{(2+1)^{k-2} - 1\} = \frac{k(k-1)}{4} \cdot (3^{k-2} - 1).$$



# PRACTICE PAPER

Exam on  
21<sup>st</sup> May  
2017

# JEE ADVANCED

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## SINGLE OPTION CORRECT TYPE

- If  $f(x) = 0$  is a cubic equation with positive and distinct roots  $\alpha, \beta, \gamma$  such that  $\beta$  is the H.M. of the roots of  $f'(x) = 0$ . Then  $\alpha, \beta, \gamma$  are in  
(a) A.P. (b) G.P.  
(c) H. P. (d) none of these
- All the roots of the equation  $11z^{10} + 10iz^9 + 10iz - 11 = 0$  lie  
(a) inside  $|z| = 1$  (b) on  $|z| = 1$   
(c) outside  $|z| = 1$  (d) none of these
- Let  $y = f(x)$  be a continuous and differentiable curve. The normals at  $(1, f(1))$ ,  $(2, f(2))$  and  $(3, f(3))$  make angles  $\pi/3, \pi/4$  and  $\pi/6$  with positive  $x$ -axis respectively. Then value of  $\int_1^2 (f(x) + xf'(x))dx + \int_2^3 f'(x)f''(x)dx$  is equal to  
(a)  $f'(2) + f(1) + 1$  (b)  $2f(2) - f(1) + 1$   
(c)  $f(3) - f(2) + 1$  (d) none of these
- $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2}$  is equal to  
(a)  $\pi$  (b)  $\pi/4$   
(c)  $\pi/2$  (d) none of these
- If  $\alpha = x + y + z + \omega$  and  $\beta = (xy + yz + z\omega + \omega x + \omega y + xz)$ , then which of the following statement(s), is/are true?  
(a)  $8\alpha^2 \geq 3\beta$  (b)  $3\alpha^2 \geq 8\beta$   
(c)  $\alpha^2\beta \geq 27$  (d) none of these
- If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{a} \cdot \vec{b} = 1$  and  $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$ , then  $\vec{b}$  is equal to  
(a)  $2\hat{i}$  (b)  $\hat{i} - \hat{j} + \hat{k}$   
(c)  $\hat{i}$  (d)  $2\hat{j} - \hat{k}$
- If  $\int_{-\pi/4}^{3\pi/4} \frac{e^{\pi/4} dx}{(e^x + e^{\pi/4})(\sin x + \cos x)} = k \int_{-\pi/2}^{\pi/2} \sec x dx$ , then the value of  $k$  is  
(a)  $\frac{1}{2}$  (b)  $\frac{1}{\sqrt{2}}$  (c)  $\frac{1}{2\sqrt{2}}$  (d)  $-\frac{1}{\sqrt{2}}$
- A line meets the co-ordinate axes at  $A$  and  $B$ . A circle is circumscribed about  $\triangle OAB$ . If the distances from  $A$  and  $B$  of the tangent to the circle at the origin be  $m$  and  $n$  respectively, then diameter of the circle is  
(a)  $m(m+n)$  (b)  $m+n$   
(c)  $n(m+n)$  (d)  $m^2 + n^2$
- If  $I_1 = \int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$  and  $I_2 = \int_{-100}^{101} \frac{dx}{5+2x-2x^2}$ , then  $\frac{I_1}{I_2}$  is  
(a)  $1/2$  (b)  $2$   
(c)  $0$  (d) none of these
- Let  $f(x) = \begin{cases} |x|, & \text{for } 0 < |x| \leq 3 \\ 1, & \text{for } x = 0 \end{cases}$ , then at  $x = 0$   $f(x)$  has  
(a) a local maximum (b) no local maximum  
(c) a local minimum (d) no extremum
- If  $x^2 + 9y^2 + 25z^2 = xyz \left( \frac{15}{x} + \frac{5}{y} + \frac{3}{z} \right)$ , then  $x, y, z$  are in  
(a) A.P. (b) G.P.  
(c) A.G.P. (d) H.P.
- The set of values of ' $r$ ' for which  ${}^{23}C_r + 2 \cdot {}^{23}C_{r+1} + {}^{23}C_{r+2} \geq {}^{25}C_{15}$  contains  
(a) 3 elements (b) 4 elements  
(c) 5 elements (d) 6 elements

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13. In an equilateral triangle inradius( $r$ ), circumradius ( $R$ ) and exradius ( $r_1$ ) are in

(a) A.P. (b) G.P.  
(c) H.P. (d) none of these

14. If  $\tan A, \tan B$  are the roots of the quadratic equation  $abx^2 - c^2x + ab = 0$  where  $a, b, c$  are sides of the  $\triangle ABC$  then  $\sin^2 A + \sin^2 B + \sin^2 C$  is

(a) 1 (b) 2 (c) 3 (d)  $3/2$

15. The real function

$$f(x) = \cos^{-1} \sqrt{x^2 + 3x + 1} + \cos^{-1} \sqrt{x^2 + 3x}$$

is defined on the set

(a)  $\{0, 3\}$  (b)  $(0, 3)$   
(c)  $\{0, -3\}$  (d)  $(-3, 0)$

16. If a function satisfies  $f(x+1) + f(x-1) = \sqrt{2}f(x)$ , then the period of  $f(x)$  can be

(a) 2 (b) 4 (c) 6 (d) 8

17. If  $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n$  where  ${}^nC_0, {}^nC_1, {}^nC_2, \dots$  are binomial coefficients. Then the value of  $2({}^nC_0 + {}^nC_3 + {}^nC_6 + \dots) + ({}^nC_1 + {}^nC_4 + {}^nC_7 + \dots)(1+\omega) + ({}^nC_2 + {}^nC_5 + {}^nC_8 + \dots)(1+\omega^2)$ , where  $\omega$  is the cube root of unity and  $n$  is a multiple of 3, is equal to

(a)  $2^n + 1$  (b)  $2^{n-1} + 1$   
(c)  $2^{n+1} - 1$  (d)  $2^n - 1$

18. The chords of contact of the pair of tangents drawn from each point on the line  $2x + y = 4$  to  $x^2 + y^2 = 1$  pass through the point

(a)  $(1, 2)$  (b)  $(1/2, 1/4)$   
(c)  $(2, 4)$  (d) none of these

19. If  $\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$ , where  $a, b, c > 0$ , then family

of lines  $\sqrt{ax} + \sqrt{by} + \sqrt{c} = 0$  passes through the fixed point given by

(a)  $(1, 1)$  (b)  $(1, -2)$   
(c)  $(-1, 2)$  (d)  $(-1, 1)$

20. If  $f(x)$  is continuous such that  $|f(x)| \leq 1 \forall x \in R$  and

$$g(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}, \text{ then range of } g(x) \text{ is}$$

(a)  $[0, 1]$  (b)  $\left[0, \frac{e^2 + 1}{e^2 - 1}\right]$

(c)  $\left[0, \frac{e^2 - 1}{e^2 + 1}\right]$  (d)  $\left[\frac{1 - e^2}{1 + e^2}, 0\right]$

21. Let three non-collinear points  $A, B$  and  $C$  have position vector  $\vec{a}, \vec{b}$  and  $\vec{c}$ , then the vector  $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$  is

(a) parallel to plane containing  $\vec{a}, \vec{b}$  and  $\vec{c}$   
(b) perpendicular to the plane formed with the vectors  $\vec{AB}$  and  $\vec{AC}$   
(c) perpendicular to the plane containing  $\vec{a}, \vec{b}$  and  $\vec{c}$   
(d) none of these

22. 271 should be split into how many parts so as to maximize their product?

(a) 99 (b) 10  
(c) 105 (d) none of these

#### MORE THAN ONE OPTION CORRECT TYPE

23. If  $0 \leq \alpha, \beta \leq \frac{\pi}{2}$  and  $\cos \alpha + \cos \beta = 1$ , then

(a)  $\alpha + \beta \geq \frac{\pi}{2}$  (b)  $\cos(\alpha + \beta) \leq 0$   
(c)  $\alpha + \beta \leq \frac{\pi}{2}$  (d)  $\cos(\alpha + \beta) \geq 0$

24. If  $\lim_{x \rightarrow 0} \frac{ae^{2x} - b \cos 2x + ce^{-2x} - x \sin x}{x^2} = 1$  and

$$f(t) = (a+b)t^2 + (a-b)t + c, \text{ then}$$

(a)  $a + b + c = 1$  (b)  $a + b + c = 2$   
(c)  $f(1) = 3/4$  (d)  $f(1) = 1$

25. If the conics whose equations are

$$S_1 : (\sin^2 \theta)x^2 + (2h \tan \theta)xy + (\cos^2 \theta)y^2 + 32x + 16y + 19 = 0$$

$$S_2 : (\cos^2 \theta)x^2 + (2h' \cot \theta)xy + (\sin^2 \theta)y^2 + 16x + 32y + 19 = 0$$

intersect in four concyclic points, where  $\theta \in \left[0, \frac{\pi}{2}\right]$ ,

then the correct statement(s) can be

(a)  $\lambda = 0$  (b)  $\lambda = 0$   
(c)  $\theta = \pi/4$  (d) none of these

26.  $ABCD$  is a square of side 1 unit.  $P$  and  $Q$  lie on the side  $AB$  and  $R$  lies on the side  $CD$ . The possible values for the circumradius of triangle  $PQR$  is ?

(a) 0.5 (b) 0.6  
(c) 0.7 (d) none of these

27. If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{a} \cdot \vec{b} = 2$  and  $\vec{a} \times \vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ , then

(a)  $\vec{a} + \vec{b} = 5\hat{i} - 4\hat{j} + 2\hat{k}$   
(b)  $\vec{a} + \vec{b} = 3\hat{i} + 2\hat{k}$  (c)  $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$   
(d)  $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

28. The solution of  $x^{1/3} + (2x - 3)^{1/3} = [3(x - 1)]^{1/3}$  is

(a) 0 (b)  $3/2$   
(c) 1 (d) none of these

29. Let  $a_1, a_2, a_3, \dots, a_n$  are in G.P. such that  $3a_1 + 7a_2 + 3a_3 - 4a_5 = 0$ , then common ratio of G.P. can be  
(a) 2 (b)  $3/2$  (c)  $5/2$  (d)  $-1/2$

30. The diagonals of a square are along the pair represented by  $2x^2 - 3xy - 2y^2 = 0$ . If  $(2, 1)$  is the vertex of the square, then the other vertices are  
(a)  $(-1, 2)$  (b)  $(1, -2)$  (c)  $(-2, -1)$  (d)  $(1, 2)$

31. Let  $f(x) = \begin{cases} \int_0^x (5 + |1-t|) dt, & \text{if } x > 2 \\ 5x + 1, & \text{if } x \leq 2 \end{cases}$  then the function is  
(a) continuous at  $x = 2$   
(b) differentiable at  $x = 2$   
(c) discontinuous at  $x = 2$   
(d) not differentiable at  $x = 2$

### COMPREHENSION TYPE

Let  $ABCD$  be a parallelogram whose diagonals equations are  $AC : x + 2y = 3$ ;  $BD : 2x + y = 3$ . If length of diagonal  $AC = 4$  units and area of  $ABCD = 8$  sq. units.

32. The length of other diagonal  $BD$  is  
(a)  $\frac{10}{3}$  (b) 2 (c)  $\frac{20}{3}$  (d) none of these
33. The length of side  $AB$  is equal to  
(a)  $\frac{2\sqrt{58}}{3}$  (b)  $\frac{4\sqrt{58}}{9}$  (c)  $\frac{3\sqrt{58}}{9}$  (d)  $\frac{5\sqrt{58}}{9}$
34. The length of  $BC$  is equal to  
(a)  $\frac{2\sqrt{10}}{3}$  (b)  $\frac{4\sqrt{10}}{3}$  (c)  $\frac{8\sqrt{10}}{3}$  (d) none of these

### MATRIX MATCH TYPE

35. Match the following.

Column I		Column II	
(A)	$\frac{\sin 1}{\sin 2} - \frac{\sin 5}{\sin 6}$	P.	positive
(B)	$\tan \frac{3}{2} - \frac{9}{4}$	Q.	negative
(C)	$\lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{x} \right]$ (where $[\cdot]$ denotes the greatest integer function)	R.	1
		S.	does not exist

36. Match the following.

Column I		Column II	
(A)	The area enclosed between the curves $ x  +  y  = 2$ and $x^2 = y$ (in sq. units) is	P.	$\frac{24}{5}$
(B)	The maximum value of the function $f(x) = 3 \sin x - 4 \cos x - \left(\frac{7}{3}\right)$ will be given by	Q.	$\frac{7}{3}$
(C)	The length of common chord of two circles of radii 3 and 4 units which intersect orthogonally is	R.	$\frac{16}{3}$
(D)	The length of chord intercepted by the parabola $y^2 = 4(x + 1)$ passing through its focus and inclined at $60^\circ$ with positive $x$ -axis is	S.	$\frac{8}{3}$

### INTEGER TYPE

37. If the normal to the curve  $x = t - 1, y = 3t^2 - 6$  at the point  $(1, 6)$  make intercepts  $a$  and  $b$  on  $x$  and  $y$ -axes respectively, then the value of  $\frac{a+12b}{146}$  is \_\_\_\_\_
38. Let  $S(n)$  denotes sum of first  $n$  terms of an A.P., then the value of  $S = \lim_{n \rightarrow \infty} \sum_{r=-n}^n \frac{f(r)}{n}$ , where  $f(n) = \frac{S(3n)}{S(2n) - S(n)}$ , is \_\_\_\_\_
39. If  $f(u, v, w, x) = u^2 + v^2 + w^2 + x^2 - 2(u + v + w + x) + 10$  with  $u, w \in [-3, 3]$  and  $v, x \in [1, 3]$  then  $\max \frac{1}{23} \cdot f(u, v, w, x)$  is \_\_\_\_\_
40. If  $|\vec{a}| = 2, |\vec{b}| = 3$ , and  $|\vec{c}| = 4$  then  $\frac{|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2}{87}$  cannot exceed \_\_\_\_\_

### SOLUTIONS

1. (b):  $f(x) = (x - \alpha)(x - \beta)(x - \gamma)$   
 $\Rightarrow f'(x) = 3x^2 - 2x(\alpha + \beta + \gamma) + \alpha\beta + \beta\gamma + \gamma\alpha$   
 $\Rightarrow \beta = \frac{2\alpha_1\beta_1}{\alpha_1 + \beta_1}$  (where  $\alpha_1, \beta_1$  are the roots of  $f'(x) = 0$ )  
 $\Rightarrow \beta = \frac{2(\alpha\beta + \beta\gamma + \gamma\alpha)}{2(\alpha + \beta + \gamma)} \Rightarrow \beta^2 = \gamma\alpha$

2. (b):  $z^9(11z+10i)=11-10iz \Rightarrow z^9 = \frac{11-10iz}{11z+10i}$

$\Rightarrow |z^9| = \frac{|11i+10z|}{|11z+10i|}$

$\Rightarrow |11i+10z|^2 - |11z+10i|^2 = 21(1-|z|^2)$

$\Rightarrow$  if  $|z| < 1$ , then  $|z^9| > 1$  (not possible)

and if  $|z| > 1 \Rightarrow |z^9| < 1$  (not possible)

$\Rightarrow |z| = 1$

3. (b):  $\int_1^2 xf'(x)dx = xf(x)\Big|_1^2 - \int_1^2 f(x)dx$

So,

$\int_1^2 (f(x) + xf'(x))dx + \int_2^3 f'(x)f''(x)dx = xf(x)\Big|_1^2 + \frac{(f'(x))^2}{2}\Big|_2^3$

$= 2f(2) - 1f(1) + \frac{1}{2}[(f'(3))^2 - (f'(2))^2]$

$= 2f(2) - f(1) + \frac{1}{2}[3-1] = 2f(2) - f(1) + 1$

4. (a):  $\lim_{x \rightarrow 0} \frac{\sin(\pi(1 - \sin^2(\tan(\sin x))))}{x^2}$   
 $= \lim_{x \rightarrow 0} \frac{\sin(\pi \sin^2(\tan(\sin x)))}{\pi \sin^2(\tan(\sin x))} \left( \frac{\pi \sin^2(\tan(\sin x))}{\tan^2(\sin x)} \right)$   
 $\left( \frac{\tan^2(\sin x)}{\sin^2 x} \right) \left( \frac{\sin^2 x}{x^2} \right) = \pi$

5. (b): Since,  $3(x^2 + y^2 + z^2 + \omega^2) - 2(xy + yz + zx + z\omega + \omega x + \omega y)$   
 $= (x-y)^2 + (x-z)^2 + (x-\omega)^2 + (y-z)^2 + (y-\omega)^2 + (z-\omega)^2 \geq 0$

$\Rightarrow 3\Sigma x^2 - 2\Sigma xy \geq 0 \Rightarrow \Sigma x^2 \geq \frac{2}{3}\Sigma xy$

Now,  $(x+y+z+\omega)^2 = \Sigma x^2 + 2\Sigma xy \geq \frac{2}{3}\Sigma xy + 2\Sigma xy$

$\Rightarrow (x+y+z+\omega)^2 \geq \frac{8}{3}\Sigma xy \Rightarrow \alpha^2 \geq \frac{8}{3}\beta$

6. (c): Let  $\vec{b} = \alpha \hat{i} + \beta \hat{j} + \gamma \hat{k}$ ,

$\vec{a} \times \vec{b} = \hat{j} - \hat{k} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \alpha & \beta & \gamma \end{vmatrix} = \hat{j} - \hat{k}$

$\Rightarrow \beta - \gamma = 0, \alpha - \gamma = 1, \alpha - \beta = 1$

$\Rightarrow \beta = \gamma, \alpha = 1 + \gamma, \alpha = 1 + \beta$

Also  $\vec{a} \cdot \vec{b} = 1 \Rightarrow \alpha + \beta + \gamma = 1$

$\Rightarrow \beta + 1 + \beta + \beta = 1 \Rightarrow \beta = 0$

$\therefore \alpha = 1, \gamma = 0$

$\therefore \vec{b} = \hat{i}$

7. (c):  $I = \int_{-\pi/4}^{3\pi/4} \frac{dx}{\sqrt{2}(e^{x-\pi/4} + 1)\cos\left(x - \frac{\pi}{4}\right)}$

$= \frac{1}{\sqrt{2}} \int_{-\pi/2}^{\pi/2} \frac{dt}{(e^t + 1)\cos t}$

$I = \frac{1}{\sqrt{2}} \int_{-\pi/2}^{\pi/2} \frac{e^t dt}{(e^t + 1)\cos t}$

Adding,  $2I = \frac{1}{\sqrt{2}} \int_{-\pi/2}^{\pi/2} \sec t dt$

$\therefore I = \frac{1}{2\sqrt{2}} \int_{-\pi/2}^{\pi/2} \sec x dx$

8. (b): Circumcentre of  $\Delta OAB \equiv \left(\frac{a}{2}, \frac{b}{2}\right)$

Circumradius =  $\sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$

Equation of circle  
 $x^2 + y^2 - ax - by = 0$

$\therefore$  Equation of tangent at origin  $ax + by = 0$

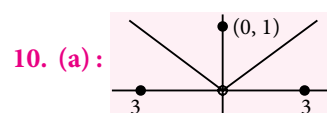
$AL = \left| \frac{a^2}{\sqrt{a^2 + b^2}} \right|, BM = \left| \frac{b^2}{\sqrt{a^2 + b^2}} \right|$

$AL + BM = m + n = \sqrt{a^2 + b^2} = \text{diameter of circle.}$

9. (a):  $I_1 = \int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$

$\Rightarrow I_1 = \int_{-100}^{101} \frac{dx}{(5+2(1-x)-2(1-x)^2)(1+e^{2-4(1-x)})}$

$\Rightarrow 2I_1 = \int_{-100}^{101} \frac{dx}{5+2x-2x^2} = I_2 \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$



10. (a):

11. (d): We have,  $x^2 + 9y^2 + 25z^2 = 15yz + 5zx + 3xy$   
 $\Rightarrow (x)^2 + (3y)^2 + (5z)^2 - (x)(3y) - (3y)(5z) - (x)(5z) = 0$

$$\Rightarrow \frac{1}{2} \left[ 2(x)^2 + 2(3y)^2 + 2(5z)^2 - 2(x)(3y) - 2(3y)(5z) - 2(x)(5z) \right] = 0$$

$$\Rightarrow \frac{1}{2} \left[ (x-3y)^2 + (3y-5z)^2 + (x-5z)^2 \right] = 0$$

$$\Rightarrow x-3y=0, 3y-5z=0, x-5z=0$$

$$\Rightarrow \frac{1}{x} = \frac{1}{3y}, \frac{5}{3y} = \frac{1}{z} \text{ and } \frac{1}{5z} = \frac{1}{x}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{1}{3y} + \frac{5}{3y} = \frac{2}{y}$$

$$\Rightarrow \therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in A.P.} \Rightarrow x, y, z \text{ are in H.P.}$$

$$\begin{aligned} 12. (d): {}^{23}C_r + 2 \cdot {}^{23}C_{r+1} + {}^{23}C_{r+2} &= {}^{24}C_{r+1} + {}^{24}C_{r+2} \\ &= {}^{25}C_{r+2} \geq {}^{25}C_{15} \end{aligned}$$

$\therefore (r+2)$  can be 10, 11, 12, 13, 14 and 15. So, 6 elements.

13. (a): In an equilateral triangle  $r = R/2$

$$\begin{aligned} \text{Also ex-radius } r_1 &= 4R \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} \\ &= 4R \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3}{2}R \end{aligned}$$

$$\Rightarrow r, R, r_1, \text{ are in A.P.}$$

$$14. (b): \text{ Here, } \tan A + \tan B = \frac{c^2}{ab} \text{ and } \tan A \cdot \tan B = 1$$

$$\tan A \cdot \tan B = 1 \Rightarrow \tan A = \cot B$$

$$\Rightarrow A = 90^\circ - B \Rightarrow A + B = 90^\circ \Rightarrow C = 90^\circ$$

$$\Rightarrow \sin A = \frac{a}{c}, \sin B = \frac{b}{c}$$

$$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \frac{a^2}{c^2} + \frac{b^2}{c^2} + 1 = 2$$

$$15. (c): \text{ Obviously, } 0 \leq x^2 + 3x + 1 \leq 1 \text{ and } 0 \leq x^2 + 3x \leq 1 \Rightarrow x^2 + 3x = 0 \Rightarrow x = 0, -3$$

16. (d): By replacing  $x = x+1$  and  $x = x-1$ , we get

$$f(x+2) + f(x) = \sqrt{2}f(x+1) \quad \dots(1)$$

$$f(x) + f(x-2) = \sqrt{2}f(x-1) \quad \dots(2)$$

Adding (1) & (2) gives,

$$\begin{aligned} f(x+2) + f(x-2) + 2f(x) &= \sqrt{2}[f(x+1) + f(x-1)] \\ &= \sqrt{2}\sqrt{2}f(x) \end{aligned}$$

$$\therefore f(x+2) + f(x-2) = 0$$

On replacing  $x$  by  $x+2$  we get  $f(x+4) + f(x) = 0$

Finally replacing  $x$  by  $x+4$ , we get

$$f(x+8) + f(x+4) = 0,$$

$$f(x+8) = -f(x+4) = f(x) \quad \forall x$$

$\therefore f(x)$  is periodic with period 8.

$$17. (d): (1+\omega)^n = C_0 + C_1\omega + C_2\omega^2 + \dots + C_n\omega^n$$

$$(1+1)^n = C_0 + C_1 + C_2 + \dots + C_n$$

$$(1+\omega)^n + (1+1)^n = 2C_0 + C_1(1+\omega) + C_2(1+\omega^2)$$

$$+ C_3(1+\omega^3) + C_4(1+\omega) + C_5(1+\omega^2) + C_6(1+\omega^3)$$

$$+ \dots + C_n(1+\omega^n)$$

$$= 2(C_0 + C_3 + C_6 + \dots) + (C_1 + C_4 + C_7 + \dots)(1+\omega)$$

$$+ (C_2 + C_5 + C_8 + \dots)(1+\omega^2) = -\omega^{2n} + 2^n$$

$$\Rightarrow 2^n - 1 \quad (\because n \text{ is a multiple of 3, } \omega^n = 1)$$

18. (b): Let  $(h, k)$  be any point on the given line

$$\therefore 2h + k = 4 \text{ and the chord of contact } hx + ky = 1$$

$$hx + (4-2h)y = 1 \Rightarrow (4y-1) + h(x-2y) = 0, p + \lambda q = 0, \text{ it passes through the intersection of } p = 0, q = 0 \text{ or } (1/2, 1/4).$$

$$19. (d): \frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}} \Rightarrow a = b + c + 2\sqrt{bc}$$

$$\Rightarrow a = (\sqrt{b} + \sqrt{c})^2 \Rightarrow (\sqrt{a} - \sqrt{b} - \sqrt{c})(\sqrt{a} + \sqrt{b} + \sqrt{c}) = 0$$

$$\Rightarrow \sqrt{a} - \sqrt{b} - \sqrt{c} = 0, \sqrt{a} + \sqrt{b} + \sqrt{c} \neq 0 (\because a, b, c > 0)$$

$$\text{Comparing with } \sqrt{a}x + \sqrt{b}y + \sqrt{c} = 0$$

$$\text{Hence } x = -1, y = 1.$$

$$20. (d): \text{ We have, } g(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}, -1 \leq f(x) \leq 1$$

$$\text{For } 0 \leq f(x) \leq 1, g(x) = 0$$

$$\text{For } -1 \leq f(x) < 0,$$

$$g(x) = \frac{e^{f(x)} - e^{-f(x)}}{e^{f(x)} + e^{-f(x)}} = \frac{e^{2f(x)} - 1}{e^{2f(x)} + 1} = 1 - \frac{2}{e^{2f(x)} + 1}$$

$$\text{For } -1 \leq f(x) < 0, g(x) \in \left[ \frac{1-e^2}{1+e^2}, 0 \right)$$

$$\text{For } -1 \leq f(x) < 1, g(x) \in \left[ \frac{1-e^2}{1+e^2}, 0 \right).$$

21. (c): Let  $\vec{r}$  be the position vector of any point on

$$\text{the plane of } \vec{a}, \vec{b} \text{ and } \vec{c} \Rightarrow (\vec{r} - \vec{a}) \cdot (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = 0$$

$$\Rightarrow (\vec{r} - \vec{a}) \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = 0$$

Thus  $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$  is perpendicular to  $\vec{r} - \vec{a}$  and thus is perpendicular to  $\vec{a}, \vec{b}$  and  $\vec{c}$ .

22. (d)

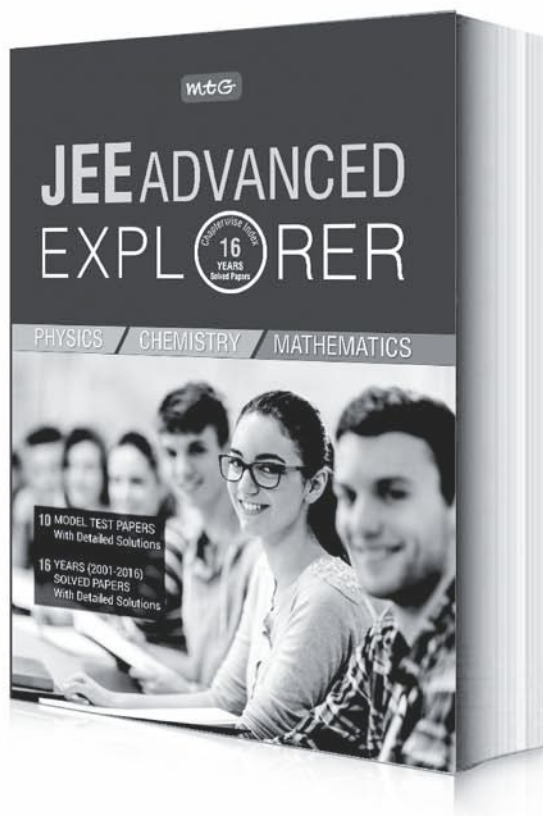
$$23. (a, b): 0 \leq \alpha, \beta \leq \frac{\pi}{2} \text{ and } \cos \alpha + \cos \beta = 1$$

$$\Rightarrow \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}$$

JEE (ADVANCED)

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$$\Rightarrow \text{at least one of } \cos\left(\frac{\alpha+\beta}{2}\right) \text{ and } \cos\left(\frac{\alpha-\beta}{2}\right) \geq \frac{1}{\sqrt{2}}$$

$$\Rightarrow \text{at least one of } \alpha+\beta \text{ and } |\alpha-\beta| \geq \frac{\pi}{2}$$

$$\text{but } |\alpha-\beta| \geq \frac{\pi}{2} \text{ and } \alpha+\beta \geq \frac{\pi}{2} \text{ and } \cos(\alpha+\beta) \leq 0$$

$$24. (a, c): \lim_{x \rightarrow 0} \frac{ae^{2x} - b \cos 2x + ce^{-2x} - x \sin x}{x^2} = 1$$

$$a \left( 1 + 2x + \frac{(2x)^2}{2!} + \dots \right) - b \left( 1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \dots \right) + c \left( 1 - 2x + \frac{(2x)^2}{2!} - \dots \right)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\dots}{x^2} = 2$$

$$\Rightarrow a - b + c = 0 \quad \dots(1)$$

$$2a - 2c = 0 \quad \dots(2)$$

$$2a - 2b + 2c = 2 \quad \dots(3)$$

$$\Rightarrow a = c \text{ and } b = 2a$$

$$\Rightarrow a + 2a + a = 1 \Rightarrow a = \frac{1}{4} = c \quad \therefore b = \frac{1}{2}$$

$$\text{Now, } f(t) = \frac{3}{4}t^2 - \frac{1}{4}t + \frac{1}{4} = \frac{3t^2 - t + 1}{4}$$

$$25. (b, c): \text{Curve through the intersection of } S_1 \text{ and } S_2 \text{ is given by } S_1 + \lambda S_2 = 0$$

$$\Rightarrow x^2(\sin^2 \theta + \lambda \cos^2 \theta) + 2(h \tan \theta - \lambda h' \cot \theta)xy + (\cos^2 \theta + \lambda \sin^2 \theta)y^2 + (32 + 16\lambda)x + (16 + 32\lambda)y + 19(1 + \lambda) = 0$$

The above equation will represent a circle if

$$\sin^2 \theta + \lambda \cos^2 \theta = \cos^2 \theta + \lambda \sin^2 \theta$$

$$\Rightarrow \sin^2 \theta - \lambda \sin^2 \theta = \cos^2 \theta - \lambda \cos^2 \theta$$

$$\Rightarrow (1 - \lambda)\sin^2 \theta = (1 - \lambda)\cos^2 \theta$$

$$\Rightarrow (1 - \lambda)(\sin^2 \theta - \cos^2 \theta) = 0 \Rightarrow \lambda = 1 \text{ or } \theta = \frac{\pi}{4}$$

$$h \tan \theta - \lambda h' \cot \theta = 0 \Rightarrow h \tan \theta = \lambda h' \cot \theta$$

which is satisfied if  $\lambda = 1$  and  $\theta = \pi/4$

$$\Rightarrow h = h'$$

26. (a, b, c): Let  $O$  be the circumcentre. Then  $OP + OR \geq PR \geq AD = 1$ , so the radius is at least  $1/2$ .  $P, Q, R$  always lie inside or on the circle through  $A, B, C, D$

which has radius  $\frac{1}{\sqrt{2}}$ , so the radius is at most  $\frac{1}{\sqrt{2}}$ .

27. (b, c): We have

$$(\vec{a} \times \vec{b}) \times \vec{a} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{b} \cdot \vec{a})\vec{a} = |\vec{a}|^2 \vec{b} - 2\vec{a}$$

$$\therefore \vec{b} = \frac{(\vec{a} \times \vec{b}) \times \vec{a} + 2\vec{a}}{|\vec{a}|^2}$$

$$\text{Now, } (\vec{a} \times \vec{b}) \times \vec{a} = 4\hat{i} - 5\hat{j} + \hat{k} \text{ and } |\vec{a}| = \sqrt{3}$$

$$\therefore \vec{b} = \frac{(4\hat{i} - 5\hat{j} + \hat{k}) + 2(\hat{i} + \hat{j} + \hat{k})}{3} = 2\hat{i} - \hat{j} + \hat{k}.$$

28. (a, b, c):  $a^{1/3} + b^{1/3} = (a + b)^{1/3}$  is true, when either  $a = 0$  or  $b = 0$  or  $a + b = 0$

$$\Rightarrow x = 0, \frac{3}{2}, 1$$

29. (b, d): Given  $3a_1 + 7a_2 + 3a_3 - 4a_5 = 0$

$$\Rightarrow 7(a_1 + a_2 + a_3) = 4(a_1 + a_3 + a_5)$$

$$\Rightarrow 7(1 + r + r^2) = 4(1 + r^2 + r^4)$$

$$\Rightarrow 7 = 4(r^2 - r + 1) \Rightarrow 4r^2 - 4r + 1 = 4$$

$$\Rightarrow (2r - 1)^2 = 4$$

$$\Rightarrow 2r - 1 = \pm 2 \Rightarrow r = \frac{3}{2}, -\frac{1}{2}$$

30. (a, b, c): Since the diagonals intersect at origin and are at right angles. Let  $B$  and  $D$  be the points adjacent to  $A$ .

$$\text{Also } |OA| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Let affix of  $B$  and  $D$  are  $z_2$  and  $z_1$  respectively.

$$\therefore \arg\left(\frac{z_2}{2+i}\right) = \frac{\pi}{2}$$

$$\Rightarrow z_2 = (2+i)i = -1 + 2i \Rightarrow B(-1, 2)$$

$$\text{Also, } \arg\left(\frac{z_1}{2+i}\right) = -\frac{\pi}{2}$$

$$\Rightarrow z_1 = (2+i)(-i) = 1 - 2i \Rightarrow D(1, -2)$$

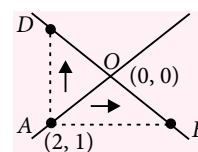
$$31. (c, d): x > 2 \int_0^x (5 + |1-t|) dt = \int_0^1 (6-t) dt + \int_1^x (4+t) dt = 1 + 4 + \frac{x^2}{2}$$

$$\Rightarrow f(x) = \begin{cases} 1 + 4x + \frac{x^2}{2}, & x > 2 \\ 5x + 1, & x \leq 2 \end{cases}$$

$$f'(x) = \begin{cases} 4 + x, & x > 2 \\ 5, & x \leq 2 \end{cases}$$

$$f(2^+) = f(2^-) = 11 \text{ continuous at } x = 2$$

$$f'(2^+) \neq f'(2^-) \Rightarrow \text{not differentiable at } x = 2$$



$$32. (c): \tan \theta = \left| \frac{-\frac{1}{2} + 2}{1+1} \right| = \frac{3}{4}, \sin \theta = \frac{3}{5},$$

$$\text{area of } \Delta CPB = \frac{1}{2} \times PC \times PB \sin \theta = 2 \Rightarrow BD = \frac{20}{3}$$

$$33. (a): \cos(\pi - \theta) = \frac{AP^2 + PB^2 - AB^2}{2AP \cdot PB}$$

$$\Rightarrow -\frac{4}{5} = \frac{4 + \frac{100}{9} - AB^2}{2 \times 2 \times \frac{10}{3}} \Rightarrow AB = \frac{2\sqrt{58}}{3}$$

$$34. (a): \text{In } \Delta CPB, \cos \theta = \frac{PC^2 + PB^2 - BC^2}{2PC \cdot PB}$$

$$\Rightarrow BC = \frac{2\sqrt{10}}{3}$$

35. (A)-(Q), (B)-(P), (C)-(S)

$$(A) \frac{\sin 1}{\sin 2} - \frac{\sin 5}{\sin 6}, \text{ Take } f(x) = \frac{\sin x}{\sin(x+1)}$$

$$f'(x) = \frac{\sin(x+1)\cos x - \cos(x+1)\sin x}{\sin^2(x+1)} = \frac{\sin 1}{\sin^2(x+1)} > 0$$

$\Rightarrow f(x)$  is increasing  $\Rightarrow f(1) < f(5)$ .

(B) Take  $f(x) = \tan x - x^2$

$$f'(x) = \sec^2 x - 2x, f''(x) = 2\sec^2 x \tan x - 2$$

$$f'(1) > 0 \text{ and } f''(x) > 0 \text{ for } x > 1$$

$$\Rightarrow f(x) \text{ is increasing in } \left(1, \frac{\pi}{2}\right) \Rightarrow \tan \frac{3}{2} > \frac{9}{4}$$

$$(C) \lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{x} \right] \text{ does not exist as left hand and right}$$

hand limits are not equal.

36. (A) - (Q), (B)-(S), (C)-(P), (D)-(R)

$$(A) \text{ Required area} = 2 \left[ \frac{1}{2} (2+1) \times 1 - \int_0^1 x^2 dx \right]$$

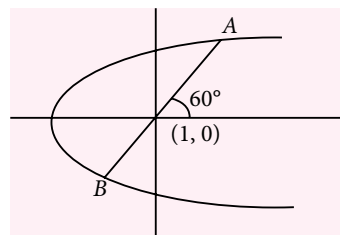
$$= 2 \left[ \frac{3}{2} - \frac{1}{3} \right] = \frac{7}{3} \text{ sq. units}$$

$$(B) 5 - \frac{7}{3} = \frac{8}{3}$$

$$(C) \text{ Length of chord} = \frac{2r_1 r_2}{\sqrt{r_1^2 + r_2^2}} = \frac{2 \times 3 \times 4}{5} = \frac{24}{5}$$

(D) Parabola is  $y^2 = 4(x+1)$  ... (i), focus is (0, 0)

$$\text{Equation of AB is } \frac{x-0}{1/2} = \frac{y-0}{\sqrt{3}/2} = r$$



Substituting parametric coordinates in (1)

$$\left( \frac{\sqrt{3}}{2} r \right)^2 = 4 \left( \frac{r}{2} + 1 \right), \frac{3r^2}{4} - 2r - 4 = 0$$

$$\begin{aligned} \text{Length of } AB &= |PA - PB| = \sqrt{(PA + PB)^2 - 4PAPB} \\ &= \sqrt{\left( \frac{8}{3} \right)^2 - 4 \times \frac{16}{3}} = \frac{16}{3} \end{aligned}$$

37. (1): Given point is corresponding to  $t = 2$  and

$$\frac{dy}{dx} = 6t \Rightarrow \text{slope of normal at } t = 2 \text{ is } -\frac{1}{12}$$

$$\therefore \text{Equation of normal is } y - 6 = -\frac{1}{12}(x - 1)$$

$$\Rightarrow a = 73, b = \frac{73}{12} \Rightarrow a + 12b = 146$$

38. (6): Let  $a$  be the first term of A.P. and  $d$  be the common difference.

$$S(3n) = \frac{3n}{2} \{2a + (3n-1)d\}, \frac{S(3n)}{S(2n) - S(n)} = 3 = f(n)$$

$$\therefore S = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=-n}^n f(r) = \lim_{n \rightarrow \infty} 3 \left( \frac{2n+1}{n} \right) = 6$$

$$39. (2): f(u, v, w, x) = (u-1)^2 + (v-1)^2 + (w-1)^2 + (x-1)^2 + 6$$

Clearly  $f$  is max when  $u = w = -3$  and  $v = x = 3$

$$\Rightarrow f_{\max} = (-3-1)^2 + (-3-1)^2 + (3-1)^2 + (3-1)^2 + 6 = 46$$

$$40. (1): |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 2(a^2 + b^2 + c^2) - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$= 2(4+9+16) - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$= 58 - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

Now  $(\vec{a} + \vec{b} + \vec{c})^2 \geq 0$

$$\Rightarrow a^2 + b^2 + c^2 \geq -2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$\Rightarrow -2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \leq 29$$

$$\Rightarrow |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 87$$





# WB JEE

## MOCK TEST PAPER

Series-7

The entire syllabus of Mathematics of WB-JEE is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

Unit No.	Topic	Syllabus In Details
UNIT NO. 7	Co-ordinate Geometry-3D	Direction ratios and direction cosines. Angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equation of a line and a plane in different forms, intersection of a line and a plane, coplanar lines.
	Differential calculus	Rolle's and Lagrange's Mean value theorem theorems, Applications of derivatives: Rate of change of quantities, monotonic-increasing and decreasing functions, Maxima and minima of functions of one variable, tangents and normals.
	Integral calculus	Integral as an anti-derivative. Fundamental integrals involving algebraic, trigonometric, exponential and logarithmic functions. Integration by substitution, by parts and by partial fractions. Integration using trigonometric identities. <b>Evaluation of simple integrals of type:</b> $\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{a^2 - x^2}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{(px + q)dx}{ax^2 + bx + c}$

Time : 1 hr 15 min.

Full marks : 50

### CATEGORY-I

For each correct answer one mark will be awarded, whereas, for each wrong answer, 25% of total marks (1/4) will be deducted. If candidates mark more than one answer, negative marking will be done.

- A straight line makes angles  $\frac{\pi}{4}$  and  $\frac{\pi}{3}$  with positive direction of  $x$ -axis and  $z$ -axis respectively. Then the acute angle made by the line with  $y$ -axis is  
(a)  $\pi/6$  (b)  $\pi/4$  (c)  $\pi/3$  (d)  $\cos^{-1}(1/3)$
- The angle between the lines  $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+4}{1}$  and  $\frac{x-3}{1} = \frac{2y+3}{5} = \frac{z-2}{2}$  is  
(a)  $\pi/3$  (b)  $\pi/2$   
(c)  $\cos^{-1}(3/5)$  (d)  $\cos^{-1}(4/5)$
- The coordinates of the foot of perpendicular drawn from the point  $A(2, 4, -1)$  on the line  $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$  are  
(a)  $(1, -3, 4)$  (b)  $(-4, 1, -3)$   
(c)  $(4, 1, 3)$  (d) none of these
- The equation of the line joining the points  $(2, -1, 4)$  and  $(1, 1, -2)$  is  
(a)  $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{6}$   
(b)  $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$   
(c)  $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{6}$   
(d)  $\frac{x-1}{-1} = \frac{y+1}{2} = \frac{z+2}{-6}$
- A plane meets the coordinate axes at  $P, Q, R$  such that the centroid of the triangle  $PQR$  is  $(a, b, c)$ . If the equation of the plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = m$ , then the value of  $m$  is  
(a) 2 (b) 3 (c) 1 (d) 6
- The equation of the plane passing through the points  $(0, 1, 0)$  and  $(3, 4, 1)$  and parallel to the line  $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$  is

By : Sankar Ghosh, S.G.M.C, Kolkata, Ph: 09831244397.

- (a)  $4x - 13y + 15z = 13$  (b)  $8x - 13y + 15z = 15$   
 (c)  $8x - 13y + 15z + 13 = 0$   
 (d) none of these
7. The vector equation of the line passing through the points  $(2, -3, 1)$  and  $(-4, 3, 6)$  is  
 (a)  $\vec{r} = -4\hat{i} + 3\hat{j} + 6\hat{k} + \lambda(2\hat{i} - 3\hat{j} + \hat{k})$   
 (b)  $\vec{r} = 2\hat{i} - 3\hat{j} + \hat{k} + \lambda(-4\hat{i} + 3\hat{j} + 6\hat{k})$   
 (c)  $\vec{r} = -6\hat{i} + 6\hat{j} + 5\hat{k} + \lambda(2\hat{i} - 3\hat{j} - 5\hat{k})$   
 (d)  $\vec{r} = 2\hat{i} - 3\hat{j} + \hat{k} + \lambda(-6\hat{i} + 6\hat{j} + 5\hat{k})$
8. If the line  $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$  is parallel to the plane  $\vec{r} \cdot (3\hat{i} - 2\hat{j} + p\hat{k}) = 4$ , then the value of  $p$  is  
 (a) 2 (b) -2 (c) 3 (d) -3
9. The vector equation of a plane passing through the point  $(1, -1, 2)$  and having 2, 3, 2 as direction number of normal to the plane is  
 (a)  $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) + 4 = 0$   
 (b)  $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) + (\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) = 0$   
 (c)  $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) + 5 = 0$   
 (d)  $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) = 3$
10. The distance of the point  $(-3, 2, 4)$  from the plane  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) + 7 = 0$  is  
 (a) 5 units (b)  $\frac{29}{7}$  units  
 (c)  $\frac{18}{7}$  units (d)  $\frac{19}{7}$  units
11. In the mean value theorem  $f(b) - f(a) = (b - a)f'(c)$  ( $a < c < b$ ), if  $a = 4$ ,  $b = 9$  and  $f(x) = \sqrt{x}$ , then the value of  $c$  is  
 (a) 8 (b) 5.25 (c) 4 (d) 6.25
12. If the function  $f(x) = 4x^3 + ax^2 + bx - 1$  satisfies all the conditions of Rolle's theorem in  $-\frac{1}{4} \leq x \leq 1$  and if  $f'\left(\frac{1}{2}\right) = 0$ , then the values of  $a$  and  $b$  are  
 (a)  $a = 2, b = -3$  (b)  $a = 1, b = -4$   
 (c)  $a = -1, b = 4$  (d)  $a = -4, b = -1$
13. Let  $f(x)$  be a continuous in  $(-\infty, \infty)$  and  $f'(x)$  exists in  $(-\infty, \infty)$ . If  $f(3) = -6$  and  $f'(x) \geq 6$  for all  $x \in [3, 6]$  then  
 (a)  $f(6) \geq 24$  (b)  $f(6) \geq 12$   
 (c)  $f(6) \leq 24$  (d)  $f(6) \leq 12$
14. A spherical balloon is being inflated at the rate of  $35 \text{ cm}^3/\text{min}$ . Then the rate of increase of surface area of the balloon when its diameter is 14 cm, is  
 (a)  $7 \text{ cm}^2/\text{min}$  (b)  $10 \text{ cm}^2/\text{min}$   
 (c)  $17.5 \text{ cm}^2/\text{min}$  (d)  $28 \text{ cm}^2/\text{min}$
15. If  $h(x) = f(x) - (f(x))^2 + (f(x))^3$  and  $f'(x) > 0 \forall x$  then  
 (a)  $h(x)$  is an increasing function of  $x$  in some specified interval  
 (b)  $h(x)$  is an increasing function  $\forall x \in R$   
 (c)  $h(x)$  is a decreasing function  $\forall x \in R$   
 (d)  $h(x)$  is a decreasing function of  $x$  in some specified interval
16. Electric current  $C$ , measured by a galvanometer, is given by the equation  $C = k \tan \theta$ , where  $k$  is a constant. Then the percentage error in the current corresponding to an error 0.7 percent in the measurement of  $\theta$  when  $\theta = 45^\circ$  is  
 (a) 1.4 (b) 2.8 (c) 1.1 (d) 2.2
17. The point on the parabola  $2y = x^2$ , which is nearest to the point  $(0, 3)$  is  
 (a)  $(\pm 4, 8)$  (b)  $\left(\pm 1, \frac{1}{2}\right)$   
 (c)  $(\pm 2, 2)$  (d)  $\left(\pm 3, \frac{9}{2}\right)$
18. If  $m$  be the slope of the tangent to the curve  $e^y = 1 + x^2$ , then  
 (a)  $|m| > 1$  (b)  $|m| < 1$   
 (c)  $m < 1$  (d)  $|m| \leq 1$
19. The normal to the curve  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$  at any point  $\theta$  is such that  
 (a) it passes through the origin  
 (b) it passes through  $(a, -a)$   
 (c) it is at a constant distance from origin  
 (d) it makes angle  $\left(\frac{\pi}{4} + \theta\right)$  with the  $x$ -axis.
20. values of  $a$  for which the function  $(a + 2)x^3 - 3ax^2 + 9ax - 1$  decreases monotonically throughout for all real values of  $x$ , are  
 (a)  $a < -2$  (b)  $a > -2$   
 (c)  $-3 < a < 0$  (d)  $a \leq -3$
21. If  $\int \frac{\sin x}{\sin(x-a)} dx = Ax + B \log |\sin(x-a)| + c$ , then the value of  $(A, B)$  is  
 (a)  $(\cos a, \sin a)$  (b)  $(-\sin a, \cos a)$   
 (c)  $(\sin a, \cos a)$  (d)  $(-\cos a, \sin a)$
22.  $\int e^{\sqrt{x}} dx$  is  
 (a)  $e^{\sqrt{x}} + c$  (b)  $2(\sqrt{x} - 1)e^{\sqrt{x}} + c$   
 (c)  $\frac{1}{2}e^{\sqrt{x}} + c$  (d)  $2(\sqrt{x} + 1)e^{\sqrt{x}} + c$

23.  $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$  is

- (a)  $\tan^{-1}x + C$  (b)  $\frac{1}{1+x^2} + C$   
 (c)  $e^{\tan^{-1}x} + C$  (d)  $\frac{2xe^{\tan^{-1}x}}{(1+x^2)^2} + C$

24.  $\int f(x)dx = f(x)$ , then  $\int \{f(x)\}^2 dx$  is equal to

- (a)  $\frac{1}{2}\{f(x)\}^2 + C$  (b)  $\{f(x)\}^3 + C$   
 (c)  $\frac{\{f(x)\}^3}{3} + C$  (d)  $\{f(x)\}^2 + C$

25. The value of  $\int \frac{dx}{2\sqrt{x(x+1)}}$  is

- (a)  $\frac{1}{2}\tan^{-1}(\sqrt{x}) + C$  (b)  $2\tan^{-1}(\sqrt{x}) + C$   
 (c)  $\tan^{-1}(\sqrt{x}) + C$  (d)  $\tan^{-1}(2\sqrt{x}) + C$

26. The value of the integral  $\int \frac{dx}{x^2+4x+13}$  is

- (a)  $\frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + C$  (b)  $\log(x^2+4x+13) + C$   
 (c)  $\frac{1}{6}\log\left|\frac{x+5}{x-1}\right| + C$  (d)  $\frac{x+2}{(x^2+4x+13)^2} + C$

27. The value of  $\int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx =$

- (a)  $e^x \sec^2 \frac{x}{2} + C$  (b)  $e^x \sec \frac{x}{2} + C$   
 (c)  $e^x \tan \frac{x}{2} + C$  (d)  $e^x \tan x + C$

28.  $I_n = \int (\log x)^n dx$ , then the value of  $(I_n + nI_{n-1})$  is

- (a)  $(\log x)^{n-1} + C$  (b)  $n(\log x)^n + C$   
 (c)  $(x \log x)^n + C$  (d)  $x(\log x)^n + C$

29.  $\int \frac{e^x dx}{(e^x+2)(e^x+1)}$  is equal to

- (a)  $\log \left| \frac{e^x+1}{e^x+2} \right| + C$  (b)  $\log \left| \frac{e^x+2}{e^x+1} \right| + C$   
 (c)  $\frac{e^x+1}{e^x+2} + C$  (d)  $\frac{e^x+2}{e^x+1} + C$

30. If  $\int \frac{dx}{\cos x + \cos \alpha} = f(\alpha) \log \left| \frac{\cos \left( \frac{x-\alpha}{2} \right)}{\cos \left( \frac{x+\alpha}{2} \right)} \right| + C$ , then

the value of  $f(\alpha)$  is

- (a)  $\sin \alpha$  (b)  $\cos \alpha$  (c)  $\operatorname{cosec} \alpha$  (d)  $\sec \alpha$

## CATEGORY-II

Every correct answer will yield 2 marks. For incorrect response, 25% of full mark (1/2) would be deducted. If candidates mark more than one answer, negative marking will be done.

31. If the lines  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ ,  $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4}$  and

$\frac{x+k}{3} = \frac{y-1}{2} = \frac{z-2}{h}$  are concurrent. Then,

- (a)  $h = -2, k = -6$  (b)  $h = 1/2, k = 2$   
 (c)  $h = 6, k = 2$  (d)  $h = 2, k = 1/2$

32. The distance between the plane whose equation is  $\vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 5$  and the line whose equation is  $\vec{r} = \hat{i} + \lambda(2\hat{i} + 5\hat{j} + 3\hat{k})$ , is

- (a)  $\frac{3}{\sqrt{14}}$  unit (b)  $\frac{5}{\sqrt{14}}$  units  
 (c) 5 units (d) none of these

33. The minimum area of the triangle formed by a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the coordinate axes, is

- (a)  $ab$  (b)  $\frac{a^2+b^2}{2}$   
 (c)  $\frac{(a+b)^2}{2}$  (d)  $\frac{a^2+ab+b^2}{3}$

34. The four common tangents to the ellipses  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and  $\frac{x^2}{4} + \frac{y^2}{9} = 1$  form

- (a) a rectangle of area 13 unit<sup>2</sup>  
 (b) a parallelogram which is neither a square nor a rectangle  
 (c) a rhombus  
 (d) none of these

35.  $\int \frac{x^2-1}{x^3 \cdot \sqrt{2x^4-2x^2+1}} dx =$

- (a)  $\frac{1}{x^2} \cdot \sqrt{2x^4-2x^2+1} + C$   
 (b)  $\frac{1}{x^3} \cdot \sqrt{2x^4-2x^2+1} + C$

$$(c) \frac{1}{x} \cdot \sqrt{2x^4 - 2x^2 + 1} + C$$

$$(d) \frac{1}{2x^2} \cdot \sqrt{2x^4 - 2x^2 + 1} + C$$

### CATEGORY-III

In this section more than 1 answer can be correct. Candidates will have to mark all the correct answers, for which 2 marks will be awarded. If, candidates marks one correct and one incorrect answer then no marks will be awarded. But if, candidate makes only correct, without making any incorrect, formula below will be used to allot marks.  $2 \times (\text{no. of correct response} / \text{total no. of correct options})$

36. Let  $\vec{A}$  be a vector parallel to the line of intersection of the planes  $P_1$  and  $P_2$  through the origin. The plane  $P_1$  is parallel to the vectors  $2\hat{j} + 3\hat{k}$  and  $4\hat{j} - 3\hat{k}$  while the plane  $P_2$  is parallel to the vectors  $\hat{j} - \hat{k}$  and  $3\hat{i} + 3\hat{j}$ . The angle between  $\vec{A}$  and  $2\hat{i} + \hat{j} - 2\hat{k}$  is  
(a)  $\pi/2$  (b)  $\pi/4$  (c)  $\pi/6$  (d)  $3\pi/4$

37.  $f(x)$  is a polynomial of third degree which has a local maxima at  $x = -1$ . If  $f(1) = -1$ ,  $f(2) = 18$  and  $f'(x)$  has a local minimum at  $x = 0$  then  
(a)  $f(0) = 5$   
(b)  $f(x)$  has local minimum at  $x = 1$   
(c)  $f(x)$  is increasing in  $[1, 2\sqrt{5}]$   
(d) The distance between  $(-1, 2)$  and  $(a, f(a))$  is  $2\sqrt{5}$ , where  $a$  is a point of local minimum.

38. A tangent to the curve  $y = f(x)$  at  $P(x, y)$  cuts the  $x$ -axis and  $y$ -axis at  $A$  and  $B$ , respectively, such that  $BP : AP = 3 : 1$ . If  $f(1) = 1$  then

- (a) the equation of the curve is  $x \frac{dy}{dx} + 3y = 0$   
(b) the curve passes through  $\left(2, \frac{1}{8}\right)$   
(c) the equation of the curve is  $x \frac{dy}{dx} - 3y = 0$   
(d) the normal at  $(1, 1)$  is  $x + 3y = 4$

39. If  $\int \frac{11 \cos x - 16 \sin x}{2 \cos x + 5 \sin x} dx = -\lambda x + \mu \log |\lambda \cos x + \delta \sin x| + C$ , then

- (a)  $\lambda = 2$  (b)  $\mu = 3$   
(c)  $\delta = \lambda + \mu$  (d)  $\delta = \mu - \lambda$

40. If  $\phi(x) = \int \frac{e^{2 \tan^{-1} x} (1+x)^2}{1+x^2} dx = x e^{m \tan^{-1} x} + C$  then

- (a)  $m = 2$  (b)  $\phi'(0) = 1$

$$(c) \phi'(1) = m e^m \quad (d) \phi'(\sqrt{3}) = \left( \frac{\sqrt{3}}{m} + 1 \right) e^{\frac{m\pi}{3}}$$

### SOLUTIONS

1. (c): Let the line makes an acute angle  $\theta$  with the  $y$ -axis. Therefore, the direction cosines are

$$l = \cos \frac{\pi}{4}, m = \cos \frac{\pi}{3} \text{ and } n = \cos \theta$$

$$\text{But } l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \cos^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \left( \frac{1}{2} + \frac{1}{4} \right) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \quad [\because \theta \text{ is acute}]$$

$$\text{So, } \theta = \pi/3$$

2. (b): The given equations of the lines are

$$\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+4}{1} \text{ and } \frac{x-3}{1} = \frac{2y+3}{5} = \frac{z-2}{2}$$

Let the angle between them be  $\theta$ . Then

$$\cos \theta = \frac{3 \cdot 1 + (-2) \cdot \frac{5}{2} + 1 \cdot 2}{\sqrt{3^2 + (-2)^2 + 1^2} \sqrt{1^2 + \left(\frac{5}{2}\right)^2 + 2^2}} = 0$$

$$\text{Hence, } \theta = \pi/2$$

3. (b): The general point on the straight line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda \text{ (say)} \quad \dots (1)$$

$$\text{is } (\lambda - 5, 4\lambda - 3, -9\lambda + 6)$$

$$\text{Let the foot of perpendicular } L \equiv (\lambda - 5, 4\lambda - 3, -9\lambda + 6)$$

$$\therefore \text{ The direction ratios of } AL \text{ are } \lambda - 7, 4\lambda - 7, -9\lambda + 7$$

Since  $AL$  is perpendicular to the line (1), therefore

$$1(\lambda - 7) + 4(4\lambda - 7) - 9(-9\lambda + 7) = 0 \Rightarrow \lambda = 1$$

Thus coordinates of foot of perpendicular are  $(-4, 1, -3)$

4. (b): The equation of the line joining the points  $(2, -1, 4)$  and  $(1, 1, -2)$  is

$$\frac{x-2}{2-1} = \frac{y+1}{-1-1} = \frac{z-4}{4+2} \text{ i.e., } \frac{x-2}{1} = \frac{y+1}{-2} = \frac{z-4}{6}$$

$$\text{i.e., } \frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$$

5. (b): Let the vertices of the triangle  $PQR$  be  $P(x, 0, 0)$ ,  $Q(0, y, 0)$  and  $R(0, 0, z)$

$$\therefore a = \frac{x}{3} \Rightarrow x = 3a, b = \frac{y}{3} \Rightarrow y = 3b$$

$$\text{and } c = \frac{z}{3} \Rightarrow z = 3c.$$

Thus, the equation of the plane is

$$\frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$$

**6. (c) :** The equation of the plane passing through the point (0, 1, 0) is  $ax + b(y - 1) + cz = 0$  ... (1)

Since (1) is also passing through the point (3, 4, 1)

$$\therefore 3a + 3b + c = 0 \quad \dots (2)$$

Again the plane (1) is parallel to the line

$$\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$$

$$\therefore 2a + 7b + 5c = 0 \quad \dots (3)$$

Now solving (2) and (3), we get

$$\frac{a}{15-7} = \frac{b}{2-15} = \frac{c}{21-6} \Rightarrow \frac{a}{8} = \frac{b}{-13} = \frac{c}{15} = \lambda \text{ (say)}$$

$$\therefore a = 8\lambda, b = -13\lambda, c = 15\lambda$$

Now (1) becomes

$$8\lambda x - 13\lambda y + 15\lambda z = -13\lambda$$

$\Rightarrow 8x - 13y + 15z + 13 = 0$  is the required equation of plane

**7. (d) :** The vector equation of the line passing through the points (2, -3, 1) and (-4, 3, 6)

$$\text{i.e. } \vec{a} = 2\hat{i} - 3\hat{j} + \hat{k} \text{ and } \vec{b} = -4\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\text{is } \vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$\Rightarrow \vec{r} = 2\hat{i} - 3\hat{j} + \hat{k} + \lambda(-6\hat{i} + 6\hat{j} + 5\hat{k})$$

**8. (b) :** Here the given equation of the line is

$$\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k}) \quad \dots (1)$$

And equation of the plane is  $\vec{r} \cdot (3\hat{i} - 2\hat{j} + p\hat{k}) = 4$  ... (2)

Since the line (1) is parallel to the plane (2)

$$\therefore 2(3) + 1(-2) + 2 \times p = 0 \Rightarrow p = -2$$

**9. (d) :** The position vector of the point having coordinates (1, -1, 2) is  $\hat{i} - \hat{j} + 2\hat{k}$  and normal to the plane is  $2\hat{i} + 3\hat{j} + 2\hat{k}$

Thus, the equation of the plane is

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) = (\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 3\hat{j} + 2\hat{k})$$

$$\text{i.e. } \vec{r} \cdot (2\hat{i} + 3\hat{j} + 2\hat{k}) = 3$$

**10. (d) :** The position vector of the point having coordinates (-3, 2, 4) is  $-3\hat{i} + 2\hat{j} + 4\hat{k}$

$$\text{Let } \vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k} \text{ and } \vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

Then the distance of the point (-3, 2, 4) from the plane

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) + 7 = 0 \text{ is}$$

$$\frac{|\vec{a} \cdot \vec{n} + 7|}{|\vec{n}|} = \frac{|(-3\hat{i} + 2\hat{j} + 4\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) + 7|}{\sqrt{2^2 + (-3)^2 + 6^2}}$$

$$= \frac{|-6 - 6 + 24 + 7|}{\sqrt{4 + 9 + 36}} = \frac{19}{7} \text{ units}$$

**11. (d) :** Here  $f(x) = \sqrt{x}$

$$\Rightarrow f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(c) = \frac{1}{2\sqrt{c}}$$

Now by mean value theorem, we have

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \frac{1}{2\sqrt{c}} = \frac{f(9) - f(4)}{9 - 4} = \frac{3 - 2}{5} = \frac{1}{5}$$

$$\Rightarrow 2\sqrt{c} = 5 \Rightarrow 4c = 25 \Rightarrow c = \frac{25}{4} = 6.25 \quad (4 < 6.25 < 9)$$

**12. (b) :** Here, the given function is

$$f(x) = 4x^3 + ax^2 + bx - 1$$

$$\Rightarrow f'(x) = 12x^2 + 2ax + b$$

$$\text{Given that } f'\left(\frac{1}{2}\right) = 0 \Rightarrow 3 + a + b = 0$$

$$\Rightarrow a + b = -3 \quad \dots (1)$$

Since  $f(x)$  satisfies all the conditions of Rolle's Theorem

$$\therefore f(1) = f\left(-\frac{1}{4}\right) \Rightarrow 4 + a + b - 1 = -\frac{1}{16} + \frac{a}{16} - \frac{b}{4} - 1$$

$$\Rightarrow a - \frac{a}{16} + b + \frac{b}{4} = -\frac{65}{4} \Rightarrow \frac{15}{16}a + \frac{5}{4}b = -\frac{65}{16}$$

$$\Rightarrow 3a + 4b = -13 \quad \dots (2)$$

Solving (1) and (2) we get  $a = 1$  and  $b = -4$

**13. (b) :** Given that  $f(x)$  is continuous in  $(-\infty, \infty)$  and  $f'(x)$  exists in  $(-\infty, \infty)$

$$\therefore f'(c) = \frac{f(6) - f(3)}{6 - 3} \text{ where } c \in (3, 6)$$

$$\Rightarrow \frac{f(6) - f(3)}{3} \geq 6 \Rightarrow f(6) - (-6) \geq 18 \Rightarrow f(6) \geq 12$$

**14. (b) :** Let the volume of the spherical balloon be  $V$

$$\therefore V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\text{Again } S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\text{Now } \frac{\frac{dS}{dt}}{\frac{dV}{dt}} = \frac{2}{r} \Rightarrow \frac{dS}{dt} = \frac{2}{r} \cdot \frac{dV}{dt} = \frac{2}{7} \times 35 = 10$$

Thus, the rate of increase of the surface area is  $10 \text{ cm}^2/\text{min}$

$$\begin{aligned}
 15. (b) : \because h(x) &= f(x) - (f(x))^2 + (f(x))^3 \\
 \therefore h'(x) &= f'(x) - 2f(x)f'(x) + 3(f(x))^2 f'(x) \\
 &= 3f'(x) \left[ (f(x))^2 - 2f(x) \cdot \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \frac{1}{3} - \frac{1}{9} \right] \\
 &= 3f'(x) \left[ \left(f(x) - \frac{1}{3}\right)^2 + \frac{2}{3} \right]
 \end{aligned}$$

It is given that  $f'(x) > 0 \forall x$

Thus  $h'(x) > 0$  for all real values of  $x$ .

Therefore  $h(x)$  is an increasing function for all real values of  $x$ .

16. (c) : We have,  $C = k \tan \theta$

$$\Rightarrow \frac{dC}{d\theta} = k \sec^2 \theta \Rightarrow \frac{dC}{d\theta} \delta \theta = k \sec^2 \theta \delta \theta$$

$$\Rightarrow \delta C = k \sec^2 \theta \delta \theta \left[ \because \delta C = \frac{dC}{d\theta} \delta \theta \right]$$

$$\text{Now, } \frac{\delta C}{C} \times 100 = \frac{k \sec^2 \theta \delta \theta}{k \tan \theta} \times 100 = \frac{\sec^2 \theta}{\tan \theta} \delta \theta \times 100$$

$$= 2 \times \frac{\pi}{4} \frac{\delta \theta}{\theta} \times 100 = \frac{\pi}{2} \times 0.7 = 1.57 \times 0.7 = 1.099$$

$\therefore$  Percentage error in  $C$  is 1.1

17. (c) : Let the point on the parabola  $2y = x^2$  be  $\left(x, \frac{x^2}{2}\right)$

Now the distance between the point  $\left(x, \frac{x^2}{2}\right)$  and  $(0, 3)$  be  $L$  (say)

$$\text{So, } L^2 = x^2 + \left(\frac{x^2}{2} - 3\right)^2$$

$$\begin{aligned}
 \frac{dL^2}{dx} &= 2x + 2\left(\frac{x^2}{2} - 3\right) \times \frac{1}{2} \times 2x \\
 &= 2x \left(1 + \frac{x^2}{2} - 3\right) = 2x \left(\frac{x^2}{2} - 2\right)
 \end{aligned}$$

$$\frac{d^2(L^2)}{dx^2} = 3x^2 - 4$$

$$\text{Now } \frac{dL^2}{dx} = 0 \text{ gives } x^3 - 4x = 0$$

$$\Rightarrow x(x^2 - 4) = 0 \Rightarrow x = 0 \text{ or } x = \pm 2$$

$$\therefore \left. \frac{d^2(L^2)}{dx^2} \right|_{x=\pm 2} = 8 > 0$$

So  $L^2$  is minimum when  $x = \pm 2$

Therefore the required point is  $(\pm 2, 2)$ .

18. (d) : The given equation of the curve is

$$e^y = 1 + x^2 \quad \dots (1)$$

Differentiating both sides w.r.t.  $x$ , we get

$$e^y \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{2x}{e^y} = \frac{2x}{1+x^2} = m \text{ (using (1))}$$

$$\Rightarrow |m| = \frac{2|x|}{1+|x|^2} \quad \dots (2)$$

$$\text{But } (1 - |x|)^2 \geq 0 \Rightarrow 1 + |x|^2 - 2|x| \geq 0$$

$$\Rightarrow 1 + |x|^2 \geq 2|x|$$

Thus, from (2) we get  $|m| \leq 1$

19. (c) : Given that  $x = a(\cos \theta + \theta \sin \theta)$

$$\Rightarrow \frac{dx}{d\theta} = a(-\sin \theta + \sin \theta + \theta \cos \theta) = a\theta \cos \theta$$

and  $y = a(\sin \theta - \theta \cos \theta)$

$$\Rightarrow \frac{dy}{d\theta} = a(\cos \theta - \cos \theta + \theta \sin \theta) = a\theta \sin \theta$$

$$\therefore \frac{dx}{dy} = \frac{\cos \theta}{\sin \theta}$$

Therefore, the equation of the normal to the given curve at  $\theta$  is

$$y - a(\sin \theta - \theta \cos \theta) = -\frac{\cos \theta}{\sin \theta} \{x - a(\cos \theta + \theta \sin \theta)\}$$

$$\Rightarrow x \cos \theta + y \sin \theta = a(\cos^2 \theta + \theta \sin \theta \cos \theta + \sin^2 \theta - \theta \sin \theta \cos \theta)$$

$$\Rightarrow x \cos \theta + y \sin \theta = a$$

$$\Rightarrow x \cos \theta + y \sin \theta - a = 0 \quad \dots (1)$$

Now the distance of the normal from the origin is

$$\frac{|0 - a|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = a(\text{constant})$$

Therefore the normal (1) is at a constant distance from the origin.

20. (d) : Given that  $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$

$$\therefore f'(x) = 3(a+2)x^2 - 6ax + 9a$$

$$= 3(a+2) \left[ x^2 - 2 \cdot x \cdot \frac{a}{a+2} + \left(\frac{a}{a+2}\right)^2 - \frac{a^2}{(a+2)^2} \right] + 9a$$

$$= 3(a+2) \left( x - \frac{a}{a+2} \right)^2 + 9a - \frac{3a^2}{(a+2)}$$

$$= 3(a+2) \left( x - \frac{a}{a+2} \right)^2 + \frac{6a(a+3)}{a+2}$$

Given that the function decreases monotonically throughout for all real values of  $x$ .

$$\therefore f'(x) < 0 \Rightarrow 3(a+2)\left(x - \frac{a}{a+2}\right)^2 + \frac{6a(a+3)}{a+2} < 0$$

Clearly the above condition is satisfied for all real values of  $x$  when  $a \leq -3$

**21. (a) :** Let  $I = \int \frac{\sin x}{\sin(x-a)} dx$

Put  $x - a = z \Rightarrow dx = dz$

$$\begin{aligned} \therefore I &= \int \frac{\sin(z+a)}{\sin z} dz = \int \frac{\sin z \cos a + \cos z \sin a}{\sin z} dz \\ &= \cos a \int dz + \sin a \int \cot z dz = z \cos a + \sin a \cdot \log(\sin z) + c \\ &= (x-a) \cos a + \sin a \cdot \log|\sin(x-a)| + c \\ &= x \cos a + \sin a \cdot \log|\sin(x-a)| + k \quad (\text{where } k = c - a \cos a) \\ \therefore (A, B) &= (\cos a, \sin a) \end{aligned}$$

**22. (b) :** Let  $I = \int e^{\sqrt{x}} dx$

Put  $\sqrt{x} = z \Rightarrow \frac{1}{2\sqrt{x}} dx = dz \Rightarrow dx = 2z dz$

$$\therefore I = \int 2ze^z dz = 2(ze^z - e^z) + c = 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$$

**23. (c) :**  $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$  put  $\tan^{-1}x = z \Rightarrow \frac{1}{1+x^2} dx = dz$

$$\therefore I = \int e^z dz = e^z + C = e^{\tan^{-1}x} + C$$

**24. (a) :**  $\because \int f(x) dx = f(x) \therefore f(x) = f'(x)$

$$\begin{aligned} \text{Now } \int \{f(x)\}^2 dx &= \int f(x)f(x) dx = \int f(x)f'(x) dx \\ &= \int f(x) d\{f(x)\} = \frac{1}{2} \{f(x)\}^2 + C \end{aligned}$$

**25. (c) :** Let  $I = \int \frac{dx}{2\sqrt{x}(x+1)}$

Put  $\sqrt{x} = z \Rightarrow dx = 2z dz$

$$\therefore I = \int \frac{2z}{2z(z^2+1)} dz = \int \frac{dz}{1+z^2} = \tan^{-1} z + C = \tan^{-1} \sqrt{x} + C$$

**26. (a) :** We have,  $\int \frac{dx}{x^2+4x+13} = \int \frac{dx}{x^2+4x+4+9}$

$$= \int \frac{dx}{(x+2)^2+3^2} = \frac{1}{3} \tan^{-1} \left( \frac{x+2}{3} \right) + C$$

**27. (c) :** We have  $\int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$

$$= \int e^x \left( \frac{1}{2\cos^2 \frac{x}{2}} + \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} \right) dx$$

$$\begin{aligned} &= \int e^x \left( \frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx \\ &= e^x \tan \frac{x}{2} + C \quad \left[ \because \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C \right] \end{aligned}$$

**28. (d) :** We have,  $I_n = \int (\log x)^n dx$

$$= x(\log x)^n - \int \left\{ n(\log x)^{n-1} \cdot \frac{1}{x} \cdot x \right\} dx$$

$$= x(\log x)^n - n \int (\log x)^{n-1} dx + C$$

$$I_n = x(\log x)^n - nI_{n-1} + C \Rightarrow I_n + nI_{n-1} = x(\log x)^n + C$$

**29. (a) :** Let  $I = \int \frac{e^x dx}{(e^x+2)(e^x+1)}$

Put  $e^x = z \Rightarrow e^x dx = dz$

$$\begin{aligned} \therefore I &= \int \frac{dz}{(z+2)(z+1)} = \int \frac{(z+2) - (z+1)}{(z+2)(z+1)} dz \\ &= \int \frac{dz}{z+1} - \int \frac{dz}{z+2} = \log|z+1| - \log|z+2| + C \\ &= \log \left| \frac{z+1}{z+2} \right| + C = \log \left| \frac{e^x+1}{e^x+2} \right| + C \end{aligned}$$

**30. (c) :** Let  $I = \int \frac{dx}{\cos x + \cos \alpha}$

Put  $\tan \frac{x}{2} = z \Rightarrow dx = \frac{2dz}{1+z^2}$

$$\begin{aligned} \therefore I &= \int \frac{2dz}{(z^2+1) \left[ \cos \alpha + \frac{1-z^2}{1+z^2} \right]} = \int \frac{2dz}{(1+z^2) \cos \alpha + 1 - z^2} \\ &= 2 \int \frac{dz}{(1+\cos \alpha) - (1-\cos \alpha)z^2} = \frac{2}{(1-\cos \alpha)} \int \frac{dz}{\left[ \cot \left( \frac{\alpha}{2} \right) \right]^2 - z^2} \end{aligned}$$

$$= \frac{1}{\sin^2 \frac{\alpha}{2}} \cdot \frac{1}{2 \cot \frac{\alpha}{2}} \log \left| \frac{1+z \tan \frac{\alpha}{2}}{1-z \tan \frac{\alpha}{2}} \right| + C$$

$$= \operatorname{cosec} \alpha \cdot \log \left| \frac{1+\tan \frac{x}{2} \tan \frac{\alpha}{2}}{1-\tan \frac{x}{2} \tan \frac{\alpha}{2}} \right| + C$$

$$= \operatorname{cosec} \alpha \log \left| \frac{\cos \left( \frac{x-\alpha}{2} \right)}{\cos \left( \frac{x+\alpha}{2} \right)} \right| + C$$

$$\therefore f(\alpha) = \operatorname{cosec} \alpha$$

**31. (d) :** The coordinates of any point on the line

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = \lambda \quad (\text{say})$$

are given by  $x = \lambda$ ,  $y = 2\lambda$  and  $z = 3\lambda$

Thus the coordinates of a general point on first line are  $(\lambda, 2\lambda, 3\lambda)$

The coordinates of any point on the line

$$\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4} = \mu \quad (\text{say})$$

are given by  $x = 3\mu + 1$ ,  $y = -\mu + 2$  and  $z = 4\mu + 3$

Thus the coordinates of a general point on second line are

$$(3\mu + 1, -\mu + 2, 4\mu + 3)$$

If the lines intersect, then they have a common point.

So, for some values of  $\lambda$  and  $\mu$ , we must have

$$\lambda = 3\mu + 1, 2\lambda = -\mu + 2 \text{ and } 3\lambda = 4\mu + 3$$

or  $\lambda - 3\mu = 1, 2\lambda + \mu = 2, 3\lambda - 4\mu = 3$

Solving two of these equations, we get  $\lambda = 1$  and  $\mu = 0$ .

So, the point of intersection of the first two lines is  $(1, 2, 3)$ . It is also lies on the third line.

$$\text{So, } \frac{1+k}{3} = \frac{2-1}{2} = \frac{3-2}{h}$$

$$\Rightarrow \frac{1+k}{3} = \frac{1}{2} \Rightarrow k = \frac{1}{2} \quad \text{and} \quad \frac{1}{2} = \frac{1}{h} \Rightarrow h = 2$$

**32. (a) :** The vector equations of the plane and the straight line are respectively

$$\vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 5 \quad \dots (1)$$

$$\text{and } \vec{r} = \hat{i} + \lambda(2\hat{i} + 5\hat{j} + 3\hat{k}) \quad \dots (2)$$

$$\text{Here, } \vec{b} \cdot \vec{n} = (2\hat{i} + 5\hat{j} + 3\hat{k}) \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 0$$

So,  $\vec{b}$  is perpendicular to  $\vec{n}$ . Hence, the given line is parallel to the given plane.

Let the required distance be  $d$

$$d = \frac{|(\hat{i} + 0\hat{j} + 0\hat{k}) \cdot (2\hat{i} + \hat{j} - 3\hat{k}) - 5|}{\sqrt{2^2 + 1^2 + 3^2}} = \frac{3}{\sqrt{14}} \text{ unit}$$

**33. (a) :** The equation of tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ at } (a\cos\theta, b\sin\theta) \text{ is } \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1.$$

The intercepts of the tangent on the axes are  $a\sec\theta$  and  $b\csc\theta$ .

$$\therefore \text{ The area of the triangle } = \frac{1}{2} \times a\sec\theta \times b\csc\theta$$

$$= \frac{ab}{\sin 2\theta} \geq ab$$

**34. (d)**

$$\text{35. (d) : Let } I = \int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx = \int \frac{\frac{1}{x^3} - \frac{1}{x^5}}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}} dx$$

$$\text{Put } 2 - \frac{2}{x^2} + \frac{1}{x^4} = z^2 \Rightarrow 4 \left( \frac{1}{x^3} - \frac{1}{x^5} \right) dx = 2z dz$$

$$\therefore I = \frac{1}{2} \int \frac{z dz}{z} = \frac{1}{2} \int dz = \frac{1}{2} z + C = \frac{1}{2} \sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + C$$

$$= \frac{1}{2x^2} \sqrt{2x^4 - 2x^2 + 1} + C$$

**36. (a, d) :** A vector along the line of intersection of two planes one of them being parallel to the vectors  $\vec{a}$  and  $\vec{b}$  and the other plane being parallel to the vectors  $\vec{c}$  and  $\vec{d}$ , is  $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ .

$$\therefore \vec{A} = \{(2\hat{j} + 3\hat{k}) \times (4\hat{j} - 3\hat{k})\} \times \{(\hat{j} - \hat{k}) \times (3\hat{i} + 3\hat{j})\}$$

$$= -18\hat{i} \times (3\hat{i} - 3\hat{j} - 3\hat{k}) = 54(\hat{k} - \hat{j})$$

Let  $\theta$  be an angle between them, then

$$\cos\theta = \left| \frac{54(\hat{k} - \hat{j}) \cdot (2\hat{i} + \hat{j} - 2\hat{k})}{54\sqrt{2} \cdot (2\hat{i} + \hat{j} - 2\hat{k})} \right| = \left| -\frac{1}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$$

So, angle  $\frac{\pi}{4}, \frac{3\pi}{4}$

**37. (b, c) :** Let  $f(x) = ax^3 + bx^2 + cx + d$

$$f(1) = -1 \Rightarrow a + b + c + d = -1$$

$$f(2) = 18 \Rightarrow 8a + 4b + 2c + d = 18$$

$$f''(0) = 0 \Rightarrow b = 0 \text{ and } f'(1) = 0$$

$$\Rightarrow 3a + 2b + c = 0$$

$$\text{Solving, } a = \frac{19}{4}, b = 0, c = -\frac{57}{4}, d = \frac{17}{2}$$

$$\therefore f(x) = \frac{1}{4}(19x^3 - 57x + 34)$$

$$f'(x) = 0 \Rightarrow 57x^2 - 57 = 0 \Rightarrow x = 1, -1.$$

$$f''(x) = 57 \times 2x, f''(-1) < 0 \text{ and } f''(1) > 0.$$

$$\therefore f(x) \text{ has a local minimum at } x = 1.$$

$$f(x) \text{ is increasing } \Rightarrow f'(x) \geq 0$$

$$\Rightarrow x^2 - 1 \geq 0 \Rightarrow x \leq -1 \text{ or } x \geq 1$$

$$\therefore \text{ In } [1, 2\sqrt{5}], f(x) \text{ is increasing.}$$

The distance of  $(-1, 2)$  from  $(1, -1)$  is not  $2\sqrt{5}$ .

**38. (a, b) :** The equation of tangent to the curve at  $P(x, y)$  is

$$Y - y = \frac{dy}{dx}(X - x) \Rightarrow X \frac{dy}{dx} - Y = x \frac{dy}{dx} - y$$

$$\Rightarrow \frac{X}{x \frac{dy}{dx} - y} + \frac{Y}{\left(y - x \frac{dy}{dx}\right) \frac{dy}{dx}} = 1$$

$$\therefore A \left( x - y \frac{dx}{dy}, 0 \right) \text{ and } B \left( 0, y - x \frac{dy}{dx} \right)$$

Now,  $\frac{BP}{AP} = \frac{3}{1}$ .

So,  $P = \left( \frac{3}{4} \left( x - y \frac{dx}{dy} \right), \frac{1}{4} \left( y - x \frac{dy}{dx} \right) \right) = (x, y)$

$$\Rightarrow \frac{1}{4} \left( y - x \frac{dy}{dx} \right) = y \Rightarrow 3y + x \frac{dy}{dx} = 0$$

$$\Rightarrow 3 \frac{dx}{x} + \frac{dy}{y} = 0 \Rightarrow 3 \int \frac{dx}{x} + \int \frac{dy}{y} = 0$$

$$\Rightarrow 3 \log x + \log y = \log c \Rightarrow x^3 y = c$$

Now,  $f(1) = 1 \Rightarrow (1, 1)$  is on it.

$$\therefore c = 1. \text{ Hence, } x^3 y = 1$$

Also  $\left( 2, \frac{1}{8} \right)$  lies on it.

The normal to  $x^3 y = 1$  at  $(1, 1)$  has the equation

$$y - 1 = \frac{-1}{\left. \frac{dy}{dx} \right|_{(1,1)}} (x - 1)$$

$$\Rightarrow y - 1 = \frac{-1}{-3} (x - 1) = x - 3y + 2 = 0$$

**39. (a, b, c) :** Let  $I = \int \frac{11 \cos x - 16 \sin x}{2 \cos x + 5 \sin x} dx$

Let,  $11 \cos x - 16 \sin x = l(5 \cos x - 2 \sin x) + m(2 \cos x + 5 \sin x)$

Now equating the coefficient of  $\cos x$  and  $\sin x$ , we get

$$5l + 2m = 11 \quad \dots (1)$$

$$\text{and } -2l + 5m = -16 \Rightarrow 2l - 5m = 16 \quad \dots (2)$$

Solving (1) and (2), we get  $l = 3$  and  $m = -2$

$$\therefore I = \int \frac{3(5 \cos x - 2 \sin x) - 2(2 \cos x + 5 \sin x)}{2 \cos x + 5 \sin x} dx$$

$$= 3 \int \frac{(5 \cos x - 2 \sin x)}{2 \cos x + 5 \sin x} dx - 2 \int dx$$

$$= 3 \log |2 \cos x + 5 \sin x| - 2x + C$$

$$= -2x + 3 \log |2 \cos x + 5 \sin x| + C$$

$$\therefore \lambda = 2, \mu = 3 \text{ and } \delta = 5$$

**40. (a, b, c, d) :** We have,  $\phi(x) = \int \frac{e^{2 \tan^{-1} x} (1+x)^2}{1+x^2} dx$

$$= \int \frac{e^{2 \tan^{-1} x} (1+x^2+2x)}{1+x^2} dx = \int e^{2 \tan^{-1} x} dx + \int \frac{2xe^{2 \tan^{-1} x}}{1+x^2} dx$$

$$= xe^{2 \tan^{-1} x} - \int 2e^{2 \tan^{-1} x} \times \frac{1}{1+x^2} x dx + \int \frac{2xe^{2 \tan^{-1} x}}{1+x^2} dx + C$$

$$= xe^{2 \tan^{-1} x} + C$$

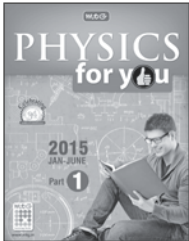
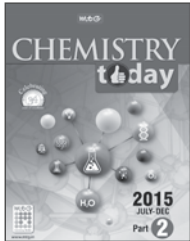
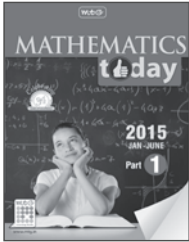
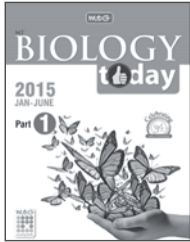
$$\therefore m = 2$$

Now  $\phi'(x) = xe^{2 \tan^{-1} x} \times \frac{2}{1+x^2} + e^{2 \tan^{-1} x}$

$$\therefore \phi'(0) = 1, \phi'(1) = 2e^{\pi/2} = me^{\pi/m}$$

$$\text{and } \phi'(\sqrt{3}) = \left( \frac{\sqrt{3}}{2} + 1 \right) e^{\frac{2\pi}{3}} = \left( \frac{\sqrt{3}}{m} + 1 \right) e^{\frac{m\pi}{3}}$$

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# MATH archives



Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of IIT-JEE Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for IIT-JEE. In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

- If  $f(x) = \frac{-x^3}{3} + x^2 \sin 6 - x \sin 4 \cdot \sin 8 - 5 \sin^{-1}(a^2 - 8a + 17)$  then  $f'(\sin 8)$  is  
(a)  $< 0$  (b)  $> 0$  (c) 12 (d) -12
- If  $\sqrt{2} < x < \sqrt{3}$ , and if  $\{x^2\} = \left\{\frac{1}{x}\right\}$ , where  $\{\cdot\}$  represents fractional part function, then the value of  $x - \frac{1}{x}$  is  
(a) -1 (b) 0 (c) 1 (d) 2
- If  $f(x) = a + b \cos^{-1} x$ , ( $b > 0$ ) has same domain and range then the value of  $a + b$  is  
(a)  $1 - \frac{\pi}{2}$  (b)  $1 - \frac{2}{\pi}$  (c)  $\frac{\pi}{2} - 1$  (d)  $\frac{2}{\pi} - 1$
- If  $x^2 + pxy + 4y^2 + x - 2y + q = 0$  represent the pair of parallel lines then  $p + q$  may be  
(a) -5 (b) 1 (c) 2 (d) 5
- Consider the circle  $x^2 + y^2 = 8$ . If from point  $P(2, 2)$ , two chords  $PA$  and  $PB$  drawn of length 1 unit then the equation of  $AB$  is  
(a)  $4x + 4y + 15 = 0$  (b)  $4x + 4y - 15 = 0$   
(c)  $x + y + 15 = 0$  (d)  $x + y - 15 = 0$
- Range of the function  $f(x) = \frac{x^3 - 1}{x - 1}$  is  
(a)  $[0, \infty)$  (b)  $\left[\frac{3}{4}, \infty\right)$   
(c)  $\left[\frac{3}{4}, \infty\right) - \{3\}$  (d)  $R$
- Let  $A$  and  $B$  be two sets containing 3 and 4 elements respectively. The number of subsets of  $A \times B$  having 2 or more elements is  
(a) 4083 (b) 4096 (c) 4046 (d) 4076
- If the vectors  $\overrightarrow{AB} = 2\hat{i} - 2\hat{k}$  and  $\overrightarrow{AC} = 4\hat{i} + \hat{j} + 2\hat{k}$  are sides of a  $\triangle ABC$ , then the length of the median through  $A$  is  
(a)  $\sqrt{\frac{33}{4}}$  (b)  $\sqrt{\frac{33}{2}}$  (c)  $\sqrt{\frac{37}{4}}$  (d)  $\sqrt{\frac{37}{2}}$
- $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5) dx =$   
(a) 3 (b) -3 (c) 0 (d) 1
- The number of all possible positive triplets  $(x, y, z)$  such that  $(2y - x)\sin\theta + (x - z)\cos^2\theta - 2x\cos\theta = 0$  for all  $\theta$  is  
(a) infinite (b) one  
(c) zero  
(d) greater than 10 but less than 100

## SOLUTIONS

- (b):  $f(x) = \frac{-x^3}{3} + x^2 \sin 6 - x \sin 4 \cdot \sin 8 - 5 \sin^{-1}(a^2 - 8a + 17)$   
 $f'(x) = -x^2 + 2x \cdot \sin 6 - \sin 4 \cdot \sin 8$   
 $f'(\sin 8) = -(\sin 8)^2 + 2 \sin 8 \cdot \sin 6 - \sin 4 \cdot \sin 8$   
 $= -\sin^2 8 + 2 \sin 8 \cdot \sin 6 - \sin 4 \cdot \sin 8$   
 $= -\sin 8 [\sin 8 + \sin 4 - 2 \sin 6]$   
 $= -\sin 8 [2 \sin 6 \cdot \cos 2 - 2 \sin 6] = -2 \sin 6 \cdot \sin 8 [\cos 2 - 1]$   
 $= 2 \sin 8 \cdot \sin 6 (1 - \cos 2) > 0$

2. (c) : Since,  $\sqrt{2} < x < \sqrt{3}$  and  $\{x^2\} = \left\{\frac{1}{x}\right\}$   
 $\Rightarrow x^2 - 2 = \frac{1}{x} \Rightarrow x^3 - 2x - 1 = 0$   
 $\Rightarrow x^3 + x^2 - x^2 - x - x - 1 = 0$   
 $\Rightarrow x^2(x+1) - x(x+1) - 1(x+1) = 0$   
 $\Rightarrow (x+1)(x^2 - x - 1) = 0$   
 $x \neq -1, x^2 - x - 1 = 0$   
 $\Rightarrow x = \frac{1 \pm \sqrt{5}}{2}$  But  $x \neq \frac{1 - \sqrt{5}}{2} \Rightarrow x = \frac{1 + \sqrt{5}}{2}$   
 $\Rightarrow \frac{1}{x} = \frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{\sqrt{5}-1}{2}$   
 $\therefore x - \frac{1}{x} = \frac{\sqrt{5}+1}{2} - \frac{\sqrt{5}-1}{2} = \frac{\sqrt{5}+1-\sqrt{5}+1}{2} = 1$

3. (d) : Domain :  $x \in [-1, 1]$  ... (i)

$\therefore 0 \leq \cos^{-1} x \leq \pi \Rightarrow 0 \leq b \cos^{-1} x \leq b\pi$  ... (ii)

$\Rightarrow a \leq a + b \cos^{-1} x \leq a + b\pi$

From (i) and (ii),  $a = -1$

$\Rightarrow a + b\pi = 1 \Rightarrow b\pi = 2 \Rightarrow b = \frac{2}{\pi}$

4. (a) : For pair of lines to be parallel,  $h^2 - ab = 0$

$\Rightarrow p = -4$

Now  $(x - 2y)^2 + (x - 2y) + q = 0$ , lines to be parallel root is real and distinct

$\Rightarrow 1 - 4q > 0 \Rightarrow q < \frac{1}{4}$

$\Rightarrow p + q = -5$

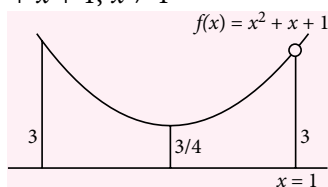
5. (b) : Required equation of AB is  $S - S' = 0$

$((x - 2)^2 + (y - 2)^2 - 1) - (x^2 + y^2 - 8) = 0$

$\Rightarrow 4x + 4y - 15 = 0$

6. (b) :  $f(x) = \frac{x^3 - 1}{x - 1} = \frac{(x - 1)(x^2 + x + 1)}{(x - 1)}$

$\Rightarrow f(x) = x^2 + x + 1, x \neq 1$



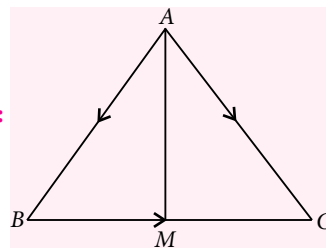
7. (a) :  $n(A \times B) = 12$

No. of subset each contain 2 or more elements is

$= ({}^{12}C_2 + {}^{12}C_3 + \dots + {}^{12}C_{12})$

$= ({}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \dots + {}^{12}C_{12}) - ({}^{12}C_0 + {}^{12}C_1)$

$= 2^{12} - (1 + 12) = 4096 - 13 = 4083$



8. (c) :

Since,  $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

$\Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} \Rightarrow \overrightarrow{BM} = \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2}$

Again,  $\overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA} = 0$

$\therefore \overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM} = \overrightarrow{AB} + \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2}$   
 $= \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = \frac{6\hat{i} + \hat{j}}{2} = 3\hat{i} + \frac{\hat{j}}{2}$

$\Rightarrow |\overrightarrow{AM}| = \sqrt{9 + \frac{1}{4}} = \sqrt{\frac{37}{4}}$

9. (c) : Put  $x - 3 = t \Rightarrow dx = dt$

$\therefore \int_1^5 (x - 1)(x - 2)(x - 3)(x - 4)(x - 5) dx$   
 $= \int_{-2}^2 (t + 2)(t + 1)t \cdot (t - 1) \cdot (t - 2) dt$   
 $= 0$  (since function is odd).

10. (c) :  $2y \sin \theta - x \sin \theta + (x - z)(1 - \sin^2 \theta) - 2x(1 - 2 \sin^2 \theta) = 0$

$\Rightarrow 2y \sin \theta - x \sin \theta + x - x \sin^2 \theta - z + z \sin^2 \theta - 2x + 4x \sin^2 \theta = 0$

$\Rightarrow (3x + z) \sin^2 \theta + (2y - x) \sin \theta - x - z = 0$

$\Rightarrow (3x + z) \sin^2 \theta + (2y - x) \sin \theta - (x + z) = 0$

This equation is zero for all values of  $\theta$ .

Thus,  $3x + z = 0, 2y - x = 0, x + z = 0$

So,  $x = \frac{-z}{3}, y = \frac{x}{2}, x = -z$

So the triplet formed is  $\left(-z, \frac{-z}{2}, z\right)$  and  $\left(\frac{-z}{3}, \frac{-z}{6}, z\right)$

Thus no positive triplet is formed.

## MPP-8 CLASS XI

## ANSWER KEY

1. (a) 2. (a) 3. (a) 4. (a) 5. (d)

6. (d) 7. (b, c) 8. (a, b) 9. (a, b, c, d)

10. (b) 11. (a, b, c) 12. (a, c) 13. (a, b, c) 14. (b)

15. (d) 16. (c) 17. (5) 18. (4) 19. (0)

20. (4)

# BRAIN @ WORK



## ELLIPSE

This article is a collection of shortcut methods, important formulas and MCQs along with their detailed solutions which provides an extra edge to the readers who are preparing for various competitive exams like JEE(Main & Advanced) and other PETs.

### DEFINITION AND STANDARD EQUATION OF ELLIPSE

An ellipse is the locus of a point in a plane which moves in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distance from a fixed straight line (called directrix) is always constant which is less than unity.

The constant ratio is generally denoted by  $e$  ( $0 < e < 1$ ) and is known as the eccentricity of the ellipse.

The general equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents an ellipse, if  $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$  and  $h^2 < ab$ .

### EQUATION OF THE ELLIPSE

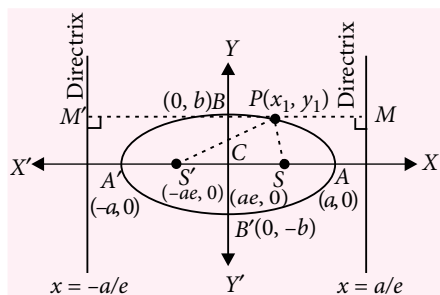
- The equation of the ellipse, whose focus is the point  $(h, k)$  and directrix is  $lx + my + n = 0$  and having eccentricity  $e$ , is

$$(x-h)^2 + (y-k)^2 = e^2 \cdot \frac{(lx + my + n)^2}{(l^2 + m^2)}$$

- Equation of the ellipse whose focus is  $(ae, 0)$  and directrix is  $x = a/e$  and having eccentricity  $e$  is given by

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1 \quad \text{or} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where  $b^2 = a^2 \cdot (1 - e^2)$



$$\text{The ellipse is } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots(1)$$

The foci  $S$  and  $S'$  are  $(ae, 0)$  and  $(-ae, 0)$ .

The equation of its directrices are  $x = a/e$  and  $x = -a/e$ .

Let  $P(x_1, y_1)$  be any point on (1),

Now,  $SP = ePM = e(a/e - x_1) = a - ex_1$

and  $S'P = ePM' = e(a/e + x_1) = a + ex_1$

$$\therefore SP + S'P = (a - ex_1) + (a + ex_1) = 2a = AA'$$

= Length of major axis

The sum of focal distances of any point on the ellipse is equal to the length of major axis.

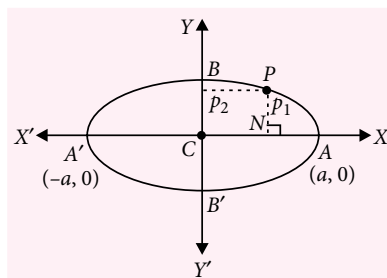
**Note :**

- Two ellipses are said to be similar, if they have the same value of eccentricity.
- Distance of every focus from the extremity of minor axis is equal to  $a$ , as  $b^2 + a^2e^2 = a^2$ .

- Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  can also be defined as

$$\Rightarrow \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2} \Rightarrow \frac{y^2}{b^2} = \frac{(a+x)(a-x)}{a^2}$$

$$\text{or } \frac{(PN)^2}{b^2} = \frac{A'N \cdot AN}{a^2} \quad \text{or} \quad \frac{(PN)^2}{AN \cdot A'N} = \frac{b^2}{a^2} = \frac{(BC)^2}{(AC)^2}$$



• **Equation of Ellipse Referred to Two Perpendicular Lines**

(a) Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  can also be defined as

$$\frac{(\text{L.P. from any point on ellipse to the major axis})^2}{(\text{Length of semi minor axis})^2} + \frac{(\text{L.P. from any point on ellipse to the minor axis})^2}{(\text{Length of semi major axis})^2} = 1$$

(where L.P. means length of perpendicular)

$$\Rightarrow \frac{p_1^2}{b^2} + \frac{p_2^2}{a^2} = 1$$

(b) If  $L_1: a_1x + b_1y + c_1 = 0$  and  $L_2: b_1x - a_1y + c_2 = 0$ , then

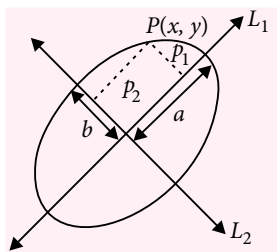
$$\text{the equation } \frac{\left( \frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} \right)^2}{b^2} + \frac{\left( \frac{b_1x - a_1y + c_2}{\sqrt{a_1^2 + b_1^2}} \right)^2}{a^2} = 1$$

represents an ellipse in the plane such that

(i) The centre of the ellipse is the point of intersection of the lines  $L_1 = 0$  and  $L_2 = 0$ .

(ii) The major axis lies along  $L_2 = 0$  and the minor axis along  $L_1 = 0$ , if  $a > b$ .

If  $a < b$ , then the major axis is along  $L_1 = 0$  and minor axis is along  $L_2 = 0$ .



(iii) If  $a > b$ , then the lengths of the major and minor axes are  $2a$  and  $2b$  respectively and if  $a < b$ , then the lengths of major and minor axes are  $2b$  and  $2a$  respectively.

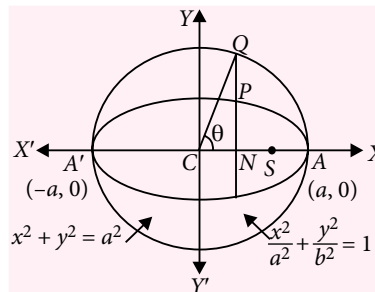
• **Parametric Equation of the Ellipse**

The circle described on the major axis of an ellipse as diameter is called the auxiliary circle of the ellipse.

Let the equation of ellipse be  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ).

$\therefore$  Equation of its auxiliary circle is  $x^2 + y^2 = a^2$   
( $\because AA'$  is diameter of the circle)

Let  $Q$  be a point on auxiliary circle  $x^2 + y^2 = a^2$  such that  $QP$  produced is perpendicular to the  $x$ -axis. Then  $P$  and  $Q$  are the corresponding points on the ellipse and the auxiliary circle respectively.



Let  $\angle QCA = \theta$  ( $0 \leq \theta < 2\pi$ )

i.e., the eccentric angle of  $P$  on an ellipse is the angle which the radius (or radius vector) through the corresponding point on the auxiliary circle makes with the major axis.

$\therefore Q \equiv (a \cos \theta, a \sin \theta)$  and  $P \equiv (a \cos \theta, b \sin \theta)$

The equation  $x = a \cos \theta$  and  $y = b \sin \theta$  taken together

are called parametric equation of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

where  $\theta$  is real parameter and  $P \equiv (a \cos \theta, b \sin \theta)$  is any point on the ellipse also known as  $P(\theta)$  or ' $\theta$ ' point on the ellipse.

**Note:**

(i) Eccentric angles of the extremities of latus

rectum of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  are given by  $\tan^{-1} \left( \pm \frac{b}{ae} \right)$ .

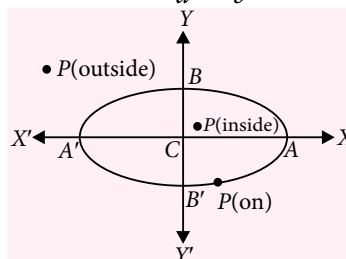
(ii) Area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\pi ab$  sq. units.

**Position of Points w.r.t. an Ellipse**

Let the ellipse be  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and  $P \equiv (h, k)$  be any

point then  $P$  will lie outside, on or inside the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  according as  $\frac{h^2}{a^2} + \frac{k^2}{b^2} >, =, < 1$



### Important Points

The equation of the ellipse whose axes are parallel to the coordinate axes and whose centre is at the origin, is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ with the following properties :}$$

S. No.	Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$
	Basic Fundamentals		
(i)	Coordinates of the centre	(0, 0)	(0, 0)
(ii)	Coordinates of the vertices	( $\pm a$ , 0)	(0, $\pm b$ )
(iii)	Coordinates of the foci	( $\pm ae$ , 0)	(0, $\pm be$ )
(iv)	Length of the major axis	$2a$	$2b$
(v)	Length of the minor axis	$2b$	$2a$
(vi)	Equation of the major axis	$y = 0$	$x = 0$
(vii)	Equation of the minor axis	$x = 0$	$y = 0$
(viii)	Equations of the directrices	$x = \pm a/e$	$y = \pm b/e$
(ix)	Eccentricity	$e = \sqrt{1 - \left(\frac{b^2}{a^2}\right)}$	$e = \sqrt{1 - \left(\frac{a^2}{b^2}\right)}$
(x)	Ends of the latus rectum	( $\pm ae$ , $\pm b^2/a$ )	( $\pm a^2/b$ , $\pm be$ )
(xi)	Length of the latus rectum	$2b^2/a$	$2a^2/b$
(xii)	Parametric coordinates	( $a \cos \theta$ , $b \sin \theta$ )	( $a \cos \theta$ , $b \sin \theta$ )
(xiii)	Auxilliary circle	$x^2 + y^2 = a^2$	$x^2 + y^2 = b^2$

### Equation of Chord

- Equation of the chord joining the points whose eccentric angles are  $\alpha$  and  $\beta$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is given by

$$\frac{x}{a} \cos \left( \frac{\alpha + \beta}{2} \right) + \frac{y}{b} \sin \left( \frac{\alpha + \beta}{2} \right) = \cos \left( \frac{\alpha - \beta}{2} \right).$$

- If the chord joining two points whose eccentric angles are  $\alpha$  and  $\beta$ , cut the major axis of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b) \text{ at the point } (ae, 0), \text{ then}$$

$$\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{e-1}{e+1}.$$

- If  $\alpha$  and  $\beta$  are the eccentric angles of extremities of a focal chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ), then

$$e = \frac{\cos \left( \frac{\alpha - \beta}{2} \right)}{\cos \left( \frac{\alpha + \beta}{2} \right)} \Rightarrow \tan \frac{\alpha}{2} \cdot \tan \frac{\beta}{2} = \frac{e-1}{e+1} \text{ or } \frac{e+1}{e-1}.$$

- Locus of mid-points of focal chords of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b) \text{ is } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \pm \frac{ex}{a}.$$

### Equation of Tangents in Different Forms

**Point Form :** Equation of tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ at the point } (x_1, y_1) \text{ is } \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1.$$

**Parametric Form :** Equation of tangent to the

$$\text{ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ at the point } (a \cos \theta, b \sin \theta)$$

$$\text{is } \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1.$$

**Slope Form :** Equation of tangent of slope  $m$  to

$$\text{the ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ are given by}$$

$y = mx \pm \sqrt{(a^2 m^2 + b^2)}$  and the coordinates of the points of contact are

$$\left( \mp \frac{a^2 m}{\sqrt{(a^2 m^2 + b^2)}}, \pm \frac{b^2}{\sqrt{(a^2 m^2 + b^2)}} \right).$$

### Results Related to Tangents

- (i) The equations of the tangents to the ellipse at points  $P(a \cos \theta_1, b \sin \theta_1)$  and  $Q(a \cos \theta_2, b \sin \theta_2)$  are  $\frac{x}{a} \cos \theta_1 + \frac{y}{b} \sin \theta_1 = 1$  and  $\frac{x}{a} \cos \theta_2 + \frac{y}{b} \sin \theta_2 = 1$  and these two intersect at the point

$$\left( \frac{a \cos \left( \frac{\theta_1 + \theta_2}{2} \right)}{\cos \left( \frac{\theta_1 - \theta_2}{2} \right)}, \frac{b \sin \left( \frac{\theta_1 + \theta_2}{2} \right)}{\cos \left( \frac{\theta_1 - \theta_2}{2} \right)} \right).$$

- (ii) The line joining two points on an ellipse, the difference of whose eccentric angle is constant, touches an other ellipse of same eccentricity.
- (iii) The locus of the point of intersection of tangents to an ellipse at the points whose eccentric angles differs by constant is an ellipse of the same eccentricity. If the eccentric angles differs by a right angle then the locus is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ .
- (iv) The locus of the point of intersection of tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at the points the sum of whose eccentric angles is constant is a straight line passing through the centre of the ellipse.
- (v) If the tangent at  $P$  on an ellipse meets the directrix in  $F$ , then  $PF$  will subtend a right angle at the corresponding focus i.e.,  $\angle PSF = \pi/2$ .
- (vi) If  $SM$  and  $S'M'$  are perpendiculars from the foci upon the tangent at any point of the ellipse, then  $SM \cdot S'M' = b^2$  and the point  $M$  &  $M'$  lie on the auxiliary circle (where  $a > b$ ).
- (vii) The tangents drawn from any point on the director circle of a given ellipse to the ellipse are always at right angle.

### Equations of Normals in Different Forms

**Point Form :** Equation of normal to the

ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at the point  $(x_1, y_1)$  is

$$\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2.$$

**Parametric Form :** Equation of normal to

the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at  $(a \cos \theta, b \sin \theta)$  is

$$ax \sec \theta - by \operatorname{cosec} \theta = a^2 - b^2.$$

**Slope Form :** Equation of normal of slope  $m$  to the

ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  are given  $y = mx \mp \frac{m(a^2 - b^2)}{\sqrt{a^2 + b^2 m^2}}$

at the points  $\left( \pm \frac{a^2}{\sqrt{a^2 + b^2 m^2}}, \pm \frac{b^2 m}{\sqrt{a^2 + b^2 m^2}} \right)$

### Results Related to Normals

- (i) If the line  $lx + my + n = 0$  be a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ then } \frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

- (ii) Locus of mid-points of normal chords of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} \right)^2 \left( \frac{a^6}{x^2} + \frac{b^6}{y^2} \right) = (a^2 - b^2)^2$$

- (iii) Four normals can be drawn from a point to an ellipse.

### Intersection of a Circle and an Ellipse

- (i) The circle  $x^2 + y^2 + 2gx + 2fy + c = 0$  and ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  can intersect each other at maximum of four points.
- (ii) The maximum number of common chord of a circle and an ellipse is six.
- (iii) The maximum number of common tangent of a circle and an ellipse is four.

### Director Circle

The locus of the point of intersection of the tangents to

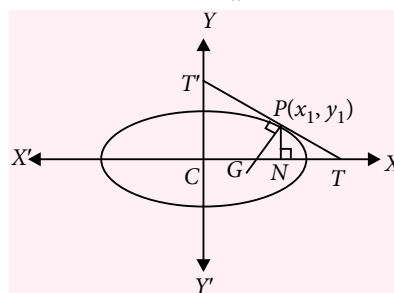
an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  which are perpendicular to each

other is called director circle and its equation is given by  $x^2 + y^2 = a^2 + b^2$ .

### Sub-tangent and Sub-normal

$$\text{Length of sub-tangent } NT = \left| \frac{a^2}{x_1} - x_1 \right|$$

$$\text{Length of sub-normal } GN = \frac{b^2}{a^2} |x_1| = (1 - e^2) |x_1|$$



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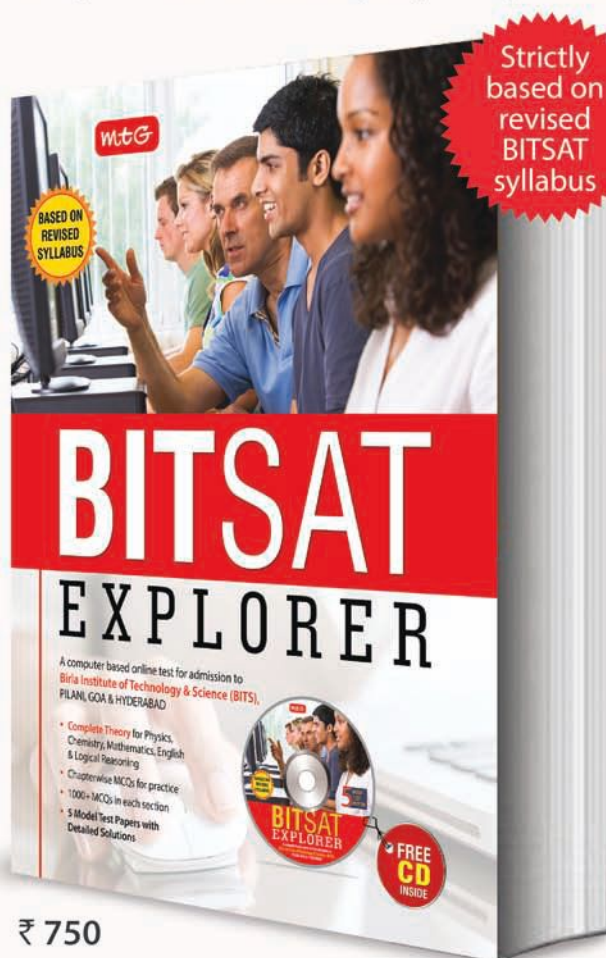


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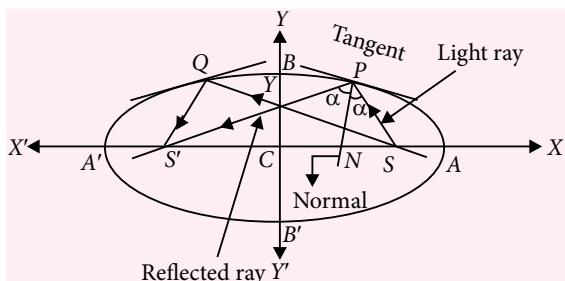
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### Reflection Property of an Ellipse

If an incoming light ray passes through the focus (S) strike the concave side of the ellipse then it will get reflected towards other focus (S')

and  $\angle SPS' = \angle SQS'$



The tangent and normal at any point of an ellipse bisect the external and internal angles between the focal radii to the point.

### PROBLEMS

1. The line  $y = mx + c$ , will be a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $|b| > |a|$ ), provided

- (a)  $c^2 = a^2 + b^2m^2$  (b)  $c^2 = b^2 + a^2m^2$   
(c)  $c^2 = (a^2 + b^2)m^2$  (d)  $c^2 = (b^2 - a^2)m^2$

2. Tangents are drawn to the ellipse  $x^2 + 2y^2 = 4$  from any arbitrary point on the line  $x + y = 4$ , the corresponding chord of contact will always pass through a fixed point, whose coordinates are

- (a)  $\left(1, \frac{1}{2}\right)$  (b)  $\left(\frac{1}{2}, 1\right)$   
(c)  $\left(1, -\frac{1}{2}\right)$  (d)  $\left(\frac{1}{2}, -1\right)$

3. Equation of the ellipse whose axes are along the coordinate axes and whose length of latus rectum and eccentricity are equal and equal to  $1/2$  each, is

- (a)  $6x^2 + 12y^2 = 1$  (b)  $12x^2 + 6y^2 = 1$   
(c)  $3x^2 + 12y^2 = 1$  (d)  $9x^2 + 12y^2 = 1$

4. The line  $y = x - 1$  touches the ellipse  $3x^2 + 4y^2 = 12$ , at

- (a)  $\left(\frac{1}{2}, -\frac{1}{2}\right)$  (b)  $(3, 2)$   
(c)  $(-1, -2)$  (d) None of these

5. One foot of normal of the ellipse  $4x^2 + 9y^2 = 36$ , that is parallel to the line  $2x + y = 3$ , is

- (a)  $\left(\frac{9}{5}, \frac{8}{5}\right)$  (b)  $\left(-\frac{9}{5}, \frac{8}{5}\right)$   
(c)  $\left(-\frac{9}{5}, -\frac{8}{5}\right)$  (d) None of these

6. Tangent drawn to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at point 'P' meets the coordinate axes at points A and B respectively. Locus of mid-point of segment AB is

- (a)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$  (b)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 2$   
(c)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$  (d)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$

7. Normals drawn to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at point 'P' meets the coordinate axes at points A and B respectively. Locus of mid point of segment AB is

- (a)  $4x^2a^2 + 4y^2b^2 = (a^2 - b^2)^2$   
(b)  $4x^2b^2 + 4y^2a^2 = (a^2 - b^2)^2$   
(c)  $16x^2a^2 + 16y^2b^2 = (a^2 - b^2)^2$   
(d)  $16x^2b^2 + 16y^2a^2 = (a^2 - b^2)^2$

8. If the line  $y = mx + c$  is a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  then corresponding point of contact is

- (a)  $\left(\frac{a^2m}{c}, \frac{b^2}{c}\right)$  (b)  $\left(-\frac{a^2m}{c}, -\frac{b^2}{c}\right)$   
(c)  $\left(\frac{a^2m}{c}, -\frac{b^2}{c}\right)$  (d)  $\left(-\frac{a^2m}{c}, \frac{b^2}{c}\right)$

9. If the line  $y = mx + c$  is a normal to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then corresponding foot of normal is

- (a)  $\left(\frac{a^2c}{m(b^2 - a^2)}, \frac{b^2c}{(b^2 - a^2)}\right)$   
(b)  $\left(\frac{a^2c}{m(a^2 - b^2)}, \frac{b^2c}{(a^2 - b^2)}\right)$   
(c)  $\left(\frac{a^2m}{c(b^2 - a^2)}, \frac{b^2c}{(b^2 - a^2)}\right)$   
(d)  $\left(\frac{a^2m}{c(a^2 - b^2)}, \frac{b^2c}{(a^2 - b^2)}\right)$

10. Locus of the mid-point of chords of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  that are parallel to the line  $y = 2x + c$ , is

- (a)  $2b^2y - a^2x = 0$  (b)  $2a^2y - b^2x = 0$   
(c)  $2b^2y + a^2x = 0$  (d)  $2a^2y + b^2x = 0$

11. The locus of the moving point  $P(x, y)$  satisfying  $\sqrt{(x-1)^2 + y^2} + \sqrt{(x+1)^2 + (y-\sqrt{12})^2} = a$

will be an ellipse if

- (a)  $a < 4$  (b)  $a > 2$  (c)  $a > 4$  (d)  $a < 2$

12. The equation  $\frac{x^2}{6-a} + \frac{y^2}{a-2} = 1$  will represent an ellipse if

- (a)  $a \in (1, 3)$  (b)  $a \in (1, 6)$   
(c)  $a \in (-\infty, 2) \cup (6, \infty)$  (d)  $a \in (2, 6) \sim \{4\}$

13. Foci of the ellipse  $16x^2 + 25y^2 = 400$  are

- (a)  $(\pm 2, 0)$  (b)  $(\pm 3, 0)$   
(c)  $\left(\pm \frac{24}{5}, 0\right)$  (d)  $\left(\pm \frac{48}{5}, 0\right)$

14. Equation of an ellipse whose focus is  $S \equiv (1, 0)$ , corresponding directrix being the line  $x + y = 2$  and eccentricity being  $\frac{1}{\sqrt{2}}$  is

- (a)  $3x^2 + 3y^2 - 2xy + 4x - 4y = 0$   
(b)  $3x^2 + 3y^2 + 2xy - 4x + 4y = 0$   
(c)  $3x^2 + 3y^2 + 2xy + 4x - 4y = 0$   
(d)  $3x^2 + 3y^2 - 2xy - 4x + 4y = 0$

15. The equation  $3(x + y - 5)^2 + 2(x - y + 7)^2 = 6$  represents an ellipse, whose centre is

- (a)  $(-1, 6)$  (b)  $(6, -1)$  (c)  $(1, -6)$  (d)  $(-6, 1)$

16. Consider an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , centered at

point 'O' and having AB and CD as its major and minor axes respectively. If  $S_1$  be one of the focus of the ellipse, radius of incircle of triangle  $OCS_1$  be 1 unit and  $OS_1 = 6$  units, then area of the ellipse is equal to

- (a)  $16\pi$  sq. units (b)  $\frac{65}{4}\pi$  sq. units  
(c)  $\frac{65}{2}\pi$  sq. units (d)  $65\pi$  sq. units

17. Normal drawn to the ellipse  $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$  at the

point  $P(\theta)$  meets the ellipse again at point  $Q(2\theta)$ , then value of  $\cos \theta$  can be

- (a)  $-\frac{16}{23}$  (b)  $-\frac{1}{2}$  (c)  $-\frac{19}{23}$  (d)  $-\frac{21}{23}$

18. P is any variable point on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

having the points  $S_1$  and  $S_2$  as its foci. Then maximum

area of the triangle  $PS_1S_2$  is equal to

- (a)  $b^2e$  sq. units (b)  $a^2e$  sq. units  
(c)  $ab$  sq. units (d)  $abe$  sq. units

19. P is any variable point on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

having the points  $S_1$  and  $S_2$  as its foci. Then Locus of incentre of triangle  $PS_1S_2$  will be

- (a) a straight line (b) a circle  
(c) a parabola (d) an ellipse

20.  $P_1$  and  $P_2$  are the foot of altitudes drawn from the foci  $S_1$  and  $S_2$  respectively of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

on one of its variable tangent. Maximum value of  $(S_1P_1)(S_2P_2)$  is equal to

- (a)  $b^2$  (b)  $b$  (c)  $a^2$  (d)  $a$

21. The equation of the ellipse centered at  $(1, 2)$  having the point  $(6, 2)$  as one of its focus and passing through the point  $(4, 6)$  is

- (a)  $\frac{(x-1)^2}{36} + \frac{3(y-2)^2}{64} = 1$   
(b)  $\frac{(x-1)^2}{18} + \frac{(y-2)^2}{32} = 1$   
(c)  $\frac{(x-1)^2}{72} + \frac{7(y-2)^2}{128} = 1$   
(d)  $\frac{(x-1)^2}{45} + \frac{(y-2)^2}{20} = 1$

22. The angle between the tangents drawn to the ellipse  $3x^2 + 2y^2 = 5$ , from the point  $P(1, 2)$  is equal to

- (a)  $\tan^{-1}(24\sqrt{5})$  (b)  $\tan^{-1}(12\sqrt{5})$   
(c)  $\tan^{-1}\left(\frac{24}{\sqrt{5}}\right)$  (d)  $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$

23. The line  $5x - 3y = 8\sqrt{2}$  is a normal to the ellipse  $\frac{x^2}{25} + \frac{y^2}{9} = 1$ . If ' $\theta$ ' be the eccentric angle of the foot of this normal then ' $\theta$ ' is equal to

- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{3}$   
(c)  $\frac{\pi}{4}$  (d) None of these

**24.** The distance of a point  $P$  (lying in the first quadrant) on the ellipse  $x^2 + 3y^2 = 6$  from the center of the ellipse is 2 units. Eccentric angle of the point ' $P$ ' is

- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{3}$   
(c)  $\frac{\pi}{4}$  (d) None of these

**25.** The tangent drawn to the ellipse  $\frac{x^2}{16} + \frac{11y^2}{256} = 1$  at the point  $P(\theta)$ , touches the circle  $(x-1)^2 + y^2 = 16$ . ' $\theta$ ' is equal to

- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{3}$   
(c)  $\frac{\pi}{4}$  (d) None of these

**26.** There are exactly two points on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  whose distance from the center of the ellipse are equal and equal to  $\sqrt{\frac{a^2 + 2b^2}{2}}$ . Eccentricity of this ellipse is equal to

- (a)  $\frac{\sqrt{3}}{2}$  (b)  $\frac{1}{\sqrt{3}}$  (c)  $\frac{1}{\sqrt{2}}$  (d)  $\sqrt{\frac{2}{3}}$

**27.** For all admissible values of the parameter ' $a$ ' the straight line  $2ax + y\sqrt{1-a^2} = 1$  will touch an ellipse whose eccentricity is equal to

- (a)  $\frac{\sqrt{3}}{2}$  (b)  $\frac{1}{\sqrt{3}}$  (c)  $\frac{1}{\sqrt{2}}$  (d)  $\sqrt{\frac{2}{3}}$

**28.** The normal to the ellipse  $4x^2 + 5y^2 = 20$  at the point ' $P$ ' touches the parabola  $y^2 = 4x$ , the eccentric angle of the point ' $P$ ' is

- (a)  $\frac{\pi}{2} + \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$  (b)  $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$   
(c)  $\pi - \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$  (d)  $\pi - \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

**29.** The tangent drawn to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , at the point ' $P$ ' meets the circle  $x^2 + y^2 = a^2$ , ( $a > b$ ) at the points  $A$  and  $B$ . The line segment  $AB$  subtends an angle  $\pi/2$  at the center of the ellipse. If ' $e$ ' be the eccentricity of the ellipse and ' $\theta$ ' be the eccentric angle of point ' $P$ ' then

- (a)  $e^2(1 + \sin^2\theta) = 1$  (b)  $e^2(1 + \cos^2\theta) = 1$   
(c)  $e^2(2 + \sin^2\theta) = 1$  (d)  $e^2(2 + \cos^2\theta) = 1$

**30.** Length of the focal chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , that is inclined at an angle ' $\theta$ ' with the  $x$ -axis, is equal to

- (a)  $\frac{2b^2a}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$  (b)  $\frac{2b^2a}{a^2 \cos^2 \theta + b^2 \sin^2 \theta}$   
(c)  $\frac{2a^2b}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$  (d)  $\frac{2a^2b}{a^2 \cos^2 \theta + b^2 \sin^2 \theta}$

**31.** The line  $y = mx + c$ , will be a normal to the ellipse,  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , if

- (a)  $\frac{a^2}{m^2} + b^2 = \frac{(b^2 - a^2)^2}{c^2}$  (b)  $\frac{c^2}{m^2} + b^2 = \frac{(b^2 - a^2)^2}{c^2}$   
(c)  $a^2 + \frac{b^2}{m^2} = \frac{(b^2 - a^2)^2}{c^2}$  (d)  $c^2 + \frac{b^2}{m^2} = \frac{(b^2 - a^2)^2}{a^2}$

**32.** Locus of the point of intersection of tangents drawn to the given ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , at points  $P(\theta_1)$

and  $Q(\theta_2)$ , such that  $\theta_1 - \theta_2 = \frac{2\pi}{3}$ , is

- (a)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$  (b)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 2$   
(c)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$  (d)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

**33.** Consider the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , having its eccentricity equal to  $e$ .  $P$  is any variable point on it and  $P_1, P_2$  are the foot of perpendiculars drawn from  $P$  to the  $x$  and  $y$ -axis respectively. The line  $P_1P_2$  will always be a normal to an ellipse whose eccentricity is equal to

- (a)  $e^2$  (b)  $\sqrt{e}$  (c)  $\sqrt{\frac{2e}{1+e}}$  (d)  $e$

**34.** The normal drawn to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) at the extremity of the latus rectum passes through the extremity of the minor axis. Eccentricity of this ellipse is equal to

(a)  $\sqrt{\frac{\sqrt{5}-1}{2}}$  (b)  $\frac{\sqrt{5}-1}{2}$

(c)  $\sqrt{\frac{\sqrt{3}-1}{2}}$  (d)  $\frac{\sqrt{3}-1}{2}$

**35.** The equation of common tangents of the curves  $x^2 + 4y^2 = 8$  and  $y^2 = 4x$  are

- (a)  $y - 2x - 4 = 0, y + 2x + 4 = 0$   
 (b)  $y - 2x - 2 = 0, y + 2x + 2 = 0$   
 (c)  $2y - x - 4 = 0, 2y + x + 4 = 0$   
 (d) None of these

**36.** If  $L$  is the length of perpendicular drawn from the origin to any normal of the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$ , then maximum value of  $L$  is

- (a) 5 (b) 4  
 (c) 1 (d) None of these

**37.** The maximum distance of the centre of the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  from the chord of contact of mutually perpendicular tangents of the ellipse is

- (a)  $\frac{9}{\sqrt{13}}$  (b)  $\frac{3}{\sqrt{13}}$  (c)  $\frac{6}{\sqrt{13}}$  (d)  $\frac{36}{\sqrt{13}}$

**38.** Tangents  $PA$  and  $PB$  are drawn to the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  from the point  $P(0, 5)$ . Area of triangle  $PAB$  is equal to

- (a)  $\frac{16}{5}$  sq. units (b)  $\frac{256}{25}$  sq. units  
 (c)  $\frac{32}{5}$  sq. units (d)  $\frac{1024}{25}$  sq. units

**39.** The straight line  $x - 2y + 4 = 0$  is one of the common tangents of the parabola  $y^2 = 4x$  and  $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$ . The equation of another common tangent of these curves is

- (a)  $x + 2y + 4 = 0$  (b)  $x + 2y - 4 = 0$   
 (c)  $x + 2y + 2 = 0$  (d)  $x + 2y - 2 = 0$

**40.** Tangent drawn to the ellipse  $\frac{x^2}{36} + \frac{y^2}{9} = 1$  at the point 'P' meets the  $y$ -axis at  $A$  and normal drawn to the ellipse at point 'P' meets the  $x$ -axis at  $B$ . If area of triangle  $OAB$  is  $\frac{27}{4}$  sq. units, then eccentric angle of point 'P' is

- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{3}$   
 (c)  $\frac{\pi}{4}$  (d) None of these

**41.** The tangent and normal drawn to the ellipse  $x^2 + 4y^2 = 4$  at the point  $P$ , meet the  $x$ -axis at  $A$  and  $B$  respectively. If  $AB = 2$  then  $\cos\theta$  is equal to, (' $\theta$ ' being the eccentric angle of point  $P$ )

- (a)  $\frac{1}{3}$  (b)  $\frac{\sqrt{8}}{3}$  (c)  $\frac{2}{3}$  (d)  $\frac{\sqrt{5}}{3}$

**42.** The chord joining the points  $P_1(a \cos\theta_1, b \sin\theta_1)$  and  $P_2(a \cos\theta_2, b \sin\theta_2)$  meets the  $x$ -axis at  $A$ . If  $OA = c$  (' $O$ ' being the origin), then the value of  $\tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2}$  is equal to

- (a)  $\frac{c-a}{c+a}$  (b)  $\frac{c-a}{c+b}$  (c)  $\frac{c-b}{c+b}$  (d)  $\frac{c-b}{c+a}$

**43.** A tangent to the ellipse  $\frac{x^2}{18} + \frac{y^2}{32} = 1$ , having slope  $-\frac{4}{3}$  cuts the  $x$  and  $y$ -axis at the points  $A$  and  $B$  respectively. If  $O$  is the origin then area of triangle  $OAB$  is equal to

- (a) 12 sq. units (b) 24 sq. units  
 (c) 36 sq. units (d) None of these

**44.** Which of the following is a common tangent to the ellipses  $\frac{x^2}{a^2+b^2} + \frac{y^2}{b^2} = 1$  and  $\frac{x^2}{a^2} + \frac{y^2}{a^2+b^2} = 1$ ?

- (a)  $by = ax - \sqrt{a^4 + a^2b^2 + b^4}$   
 (b)  $ay = bx - \sqrt{a^4 + a^2b^2 + b^4}$   
 (c)  $by = ax - \sqrt{a^4 - a^2b^2 + b^4}$   
 (d)  $ay = bx - \sqrt{a^4 - a^2b^2 + b^4}$

**45.** Maximum length of the chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , such that eccentric angles of its extremities differ by  $\frac{\pi}{2}$  is ( $a > b$ )

- (a)  $a\sqrt{2}$  (b)  $b\sqrt{2}$   
 (c)  $ab\sqrt{2}$  (d) None of these

**46.** Consider an ellipse having its axes along the coordinate axes and passing through the point  $(4, -1)$ .

If the line  $x + 4y - 10 = 0$  is one of its tangent, then area of the ellipse is equal to

- (a)  $20 \pi$  sq. units      (b)  $30 \pi$  sq. units  
(c)  $15 \pi$  sq. units      (d)  $10 \pi$  sq. units

47.  $S_1$  and  $S_2$  are the foci of an ellipse. 'B' be one of the extremity of its minor axis. If triangle  $S_1 S_2 B$  is right angled then eccentricity of the ellipse is equal to

- (a)  $\frac{\sqrt{3}}{2}$       (b)  $\frac{1}{\sqrt{2}}$   
(c)  $\sqrt{\frac{3}{2}}$       (d) None of these

48. If the chord of contact of tangents drawn from a point  $P$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  touches the circles  $x^2 + y^2 = c^2$ , then locus of  $P$  is

- (a)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2}{c^2}$       (b)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{b^4}{c^4}$   
(c)  $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$       (d)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^4}{c^4}$

49. The point on the ellipse  $4x^2 + 9y^2 = 1$  such that tangent drawn to the ellipse at this point is perpendicular to the line  $8x - 9y = 0$ , is

- (a)  $\left(\frac{27}{2\sqrt{985}}, -\frac{16}{3\sqrt{985}}\right)$       (b)  $\left(-\frac{27}{2\sqrt{985}}, \frac{16}{2\sqrt{985}}\right)$   
(c)  $\left(\frac{27}{2\sqrt{985}}, \frac{16}{3\sqrt{985}}\right)$       (d) None of these

50. Tangents drawn to the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , from the point 'P', meets the coordinate axes at concyclic points, then locus of point 'P' is

- (a)  $x^2 + y^2 = 7$       (b)  $x^2 - y^2 = 7$   
(c)  $x^2 + y^2 = 25$       (d)  $x^2 - y^2 = 25$

### SOLUTIONS

1. (b): If the point of contact is  $P(a \cos \theta, b \sin \theta)$ , then equation of tangent  $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$  and given line  $y = mx + c$  will be identical. Comparing the coefficients, we get

$$\frac{\cos \theta}{am} = \frac{\sin \theta}{-b} = -\frac{1}{c}$$

$$\Rightarrow \cos \theta = -\frac{am}{c}, \sin \theta = \frac{b}{c} \Rightarrow a^2 m^2 + b^2 = c^2$$

2. (a): Let the arbitrary point be  $(a, 4 - a)$ , then equation of corresponding chord of contact is,

$$x \cdot a + 2y(4 - a) = 4$$

$$\Rightarrow a(x - 2y) + 8y - 4 = 0$$

It is indeed a family of concurrent lines. The point of concurrency being the intersection point of the lines  $x - 2y = 0$  and  $8y - 4 = 0$

$$\therefore \text{Required point is } \left(1, \frac{1}{2}\right)$$

$$3. (d): \frac{2b^2}{a} = \frac{1}{2} \Rightarrow \frac{b^2}{a} = \frac{1}{4} \quad \dots(i) \quad (\text{Given})$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2} \quad \dots(ii)$$

From (i) and (ii), we get,  $a = \frac{1}{3}, b^2 = \frac{1}{12}$

$$\therefore \text{Equation of ellipse is } \frac{x^2}{\frac{1}{9}} + \frac{y^2}{\frac{1}{12}} = 1$$

$$\text{i.e., } 9x^2 + 12y^2 = 1$$

$$4. (d): \text{Given ellipse is, } \frac{x^2}{4} + \frac{y^2}{3} = 1$$

and line is  $y = x - 1$

Let the point of contact be  $(2 \cos \theta, \sqrt{3} \sin \theta)$ .

$$\therefore \text{Equation of tangent becomes } \frac{x}{2} \cos \theta + \frac{y}{\sqrt{3}} \sin \theta = 1$$

$$\Rightarrow \frac{\cos \theta}{2} = \frac{\sin \theta}{-\sqrt{3}} = 1$$

$$\Rightarrow \cos \theta = 2, \sin \theta = -\sqrt{3}$$

which is not possible. Thus, the given line can't be a tangent to the given ellipse.

$$5. (b): \text{Given ellipse is, } \frac{x^2}{9} + \frac{y^2}{4} = 1$$

Let the foot of normal be  $(3 \cos \theta, 2 \sin \theta)$

Equation of normal at this point is,

$$\frac{3x}{\cos \theta} - \frac{2y}{\sin \theta} = 5$$

$$\text{Slope} = \frac{3}{2} \tan \theta = -2$$

$$\Rightarrow \tan \theta = -\frac{4}{3}$$

$$\text{or } \sin \theta = \frac{4}{5}, \cos \theta = -\frac{3}{5}$$

$$\text{Thus foot of normal be } \left(-\frac{9}{5}, \frac{8}{5}\right).$$

**6. (c) :** Let  $P \equiv (a \cos \theta, b \sin \theta)$

$\therefore$  Equation of tangent is  $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$

$\Rightarrow A \equiv (a \sec \theta, 0), B \equiv (0, b \operatorname{cosec} \theta)$

If the mid-point of segment  $AB$  be  $Q(h, k)$ , then

$$2h = a \sec \theta, 2k = b \operatorname{cosec} \theta$$

$$\Rightarrow \frac{a^2}{4h^2} + \frac{b^2}{4k^2} = 1$$

$$\therefore \text{Locus of } Q \text{ is } \frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$$

**7. (a) :** Let  $P \equiv (a \cos \theta, b \sin \theta)$

$\therefore$  Equation of normal is  $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$

$$\Rightarrow A \equiv \left( \frac{a^2 - b^2}{a} \cos \theta, 0 \right) \text{ and } B \equiv \left( 0, \frac{a^2 - b^2}{b} \sin \theta \right)$$

If  $(h, k)$  be the mid-point of segment  $AB$ , then

$$2h = \frac{a^2 - b^2}{a} \cos \theta, 2k = \frac{a^2 - b^2}{b} \sin \theta$$

$$\Rightarrow 4h^2 a^2 + 4k^2 b^2 = (a^2 - b^2)^2$$

Thus, locus of the point is

$$4a^2 x^2 + 4b^2 y^2 = (a^2 - b^2)^2$$

**8. (d) :** Let the point of contact be  $P(a \cos \theta, b \sin \theta)$ .

Then equation of tangent is  $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$

Thus on comparing,  $\frac{\cos \theta}{am} = \frac{\sin \theta}{-b} = -\frac{1}{c}$

$$\Rightarrow \cos \theta = -\frac{am}{c}, \sin \theta = \frac{b}{c}$$

Thus the point of contact is  $\left( -\frac{a^2 m}{c}, \frac{b^2}{c} \right)$

**9. (a) :** Let the foot of normal be  $P(a \cos \theta, b \sin \theta)$

Equation of normal at  $P$  is  $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$

$$\therefore \frac{m \cos \theta}{a} = \frac{\sin \theta}{b} = \frac{c}{(b^2 - a^2)}$$

$$\Rightarrow \cos \theta = \frac{ac}{m(b^2 - a^2)}, \sin \theta = \frac{bc}{(b^2 - a^2)}$$

Thus the foot of normal is  $\left( \frac{a^2 c}{m(b^2 - a^2)}, \frac{b^2 c}{(b^2 - a^2)} \right)$

**10. (d) :** Let the mid-point of chord be  $P(h, k)$ . Equation of this chord will be  $T = S_1$

$$\text{i.e., } \frac{xh}{a^2} + \frac{yk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$$

$$\Rightarrow \frac{h}{2a^2} = -\frac{k}{b^2} = -\frac{\left( \frac{h^2}{a^2} + \frac{k^2}{b^2} \right)}{c} \Rightarrow k = -\frac{b^2}{2a^2} \cdot h$$

Thus the required locus is  $y = -\frac{b^2}{2a^2} x$  or  $2a^2 y + b^2 x = 0$

**11. (c) :** Let  $A \equiv (1, 0), B \equiv (-1, \sqrt{12})$

$\therefore$  Given expression is,  $PA + PB = a$

Here,  $AB = \sqrt{4+12} = 4$

Thus  $P(x, y)$  will lie on an ellipse having foci  $A$  and  $B$ , provided  $a > 4$

**12. (d) :** Clearly,  $6 - a > 0, a - 2 > 0$

and  $6 - a \neq a - 2$

$$\Rightarrow a \in (2, 6) \sim \{4\}$$

**13. (b) :** Equation of the ellipse is  $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$$\Rightarrow e^2 = 1 - \frac{16}{25} = \frac{9}{25} \Rightarrow e = \frac{3}{5}$$

Thus, foci are  $\left( 5 \cdot \frac{3}{5}, 0 \right)$  and  $\left( -5 \cdot \frac{3}{5}, 0 \right)$

i.e.  $(\pm 3, 0)$

**14. (d) :** If  $P(x, y)$  be any point on the ellipse, then  $PS = e \cdot PM$ , where  $PM$  is perpendicular distance of  $P$  from directrix.

$$\Rightarrow (x-1)^2 + y^2 = \frac{1}{2} \left( \frac{x+y-2}{\sqrt{2}} \right)^2$$

Thus required equation is  $3x^2 + 3y^2 - 2xy - 4x + 4y = 0$

**15. (a) :** Given equation can be rewritten as

$$\frac{\left( \frac{x+y-5}{\sqrt{2}} \right)^2}{1} + \frac{\left( \frac{x-y+7}{\sqrt{2}} \right)^2}{3/2} = 1$$

It is clearly an ellipse whose major and minor axes are along the lines

$$x - y + 7 = 0 \text{ and } x + y - 5 = 0$$

Thus, centre of this ellipse is the intersection point of these lines i.e., the point  $(-1, 6)$

**16. (b) :**  $OS_1 = ae = 6, OC = b$  (say),  $CS_1 = a$

$$\Delta_{OCS_1} = \frac{1}{2} \cdot (OS_1)(OC) = 3b$$

Semi perimeter of triangle  $OCS_1$

$$= \frac{1}{2} (OS_1 + OC + CS_1) = \frac{1}{2} (6 + a + b)$$

Now, inradius of triangle  $OCS_1$

$$= \frac{3b}{\frac{1}{2}(6+a+b)} = 1 \text{ (given)}$$

$$\Rightarrow 5b = 6 + a$$

$$\text{Also } b^2 = a^2(1 - e^2) = a^2 - 36$$

$$\text{Thus, } 25b^2 = 36 + a^2 + 12a$$

$$\Rightarrow 25(a^2 - 36) = 36 + a^2 + 12a$$

$$\Rightarrow 2a^2 - a - 78 = 0 \Rightarrow a = \frac{13}{2}, b = \frac{5}{2}$$

$$\text{Hence, area of ellipse} = \pi ab = \frac{65\pi}{4} \text{ sq. units.}$$

**17. (a) :** Equation of normal at  $P(\theta)$  is,

$$\frac{4x}{\cos \theta} - \frac{3y}{\sin \theta} = 7$$

Since it passes through the point  $Q(2\theta)$ , thus

$$\frac{16 \cdot \cos 2\theta}{\cos \theta} - \frac{9 \cdot \sin 2\theta}{\sin \theta} = 7$$

$$\Rightarrow \frac{16(2\cos^2 \theta - 1)}{\cos \theta} - 18 \cos \theta = 7$$

$$\Rightarrow 23 \cos^2 \theta - 7 \cos \theta - 16 = 0$$

$$\Rightarrow \cos \theta = -\frac{16}{23} \quad [\because \text{as } \cos \theta \neq 1]$$

**18. (d) :** In this case, base  $S_1S_2$  is fixed and  $PS_1 + PS_2$  is fixed. Hence area will be maximum when  $PS_1 = PS_2$

$$\therefore \text{Maximum area} = \frac{1}{2}(2ae)b = abe \text{ sq. units.}$$

**19. (d) :** Here,  $S_1S_2 = 2ae$ ,

$$PS_1 + PS_2 = 2a$$

If the normal to ellipse at  $P$ , meets the  $x$ -axis at  $Q$ , then incentre of  $\Delta PS_1S_2$  will lie on segment  $PQ$  and

$$\text{will divide it in the ratio } \frac{PS_1 + PS_2}{S_1S_2} \text{ i.e., } \frac{1}{e}$$

$$\text{Let } P \equiv (a \cos \theta, b \sin \theta)$$

Equation of normal at ' $P$ ' is

$$\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2 = a^2 e^2$$

$$\Rightarrow Q \equiv (ae^2 \cos \theta, 0)$$

Let  $R(h, k)$  be the incenter, then

$$h = \frac{a \cos \theta \cdot e + ae^2 \cos \theta}{1 + \frac{1}{e}}, k = \frac{b \sin \theta \cdot e + 0 \cdot 1}{1 + \frac{1}{e}}$$

$$\Rightarrow h \left(1 + \frac{1}{e}\right) = (ae + ae^2) \cos \theta \text{ and } be \cdot \sin \theta = k \left(1 + \frac{1}{e}\right)$$

Thus locus of  $R$  is

$$\frac{x^2 \left(1 + \frac{1}{e}\right)^2}{a^2 e^2 (1+e)^2} + \frac{y^2 \left(1 + \frac{1}{e}\right)^2}{b^2 e^2} = 1$$

which is clearly an ellipse.

**20. (a) :**  $S_1 = (ae, 0)$ ,  $S_2 = (-ae, 0)$

Let the tangent be

$$xb \cos \theta + ay \sin \theta - ab = 0$$

$$S_1P_1 = \frac{|abe \cos \theta - ab|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

$$\Rightarrow S_1P_1 = \frac{ab(1 - e \cos \theta)}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

$$\text{Similarly, } S_2P_2 = \frac{ab(1 + e \cos \theta)}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

$$\Rightarrow S_1P_1 \cdot S_2P_2 = \frac{a^2 b^2 (1 - e^2 \cos^2 \theta)}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

$$= \frac{a^2 b^2 (1 - e^2 \cos^2 \theta)}{a^2 + (b^2 - a^2) \cos^2 \theta} = \frac{a^2 b^2 (1 - e^2 \cos^2 \theta)}{a^2 - a^2 e^2 \cos^2 \theta} = b^2$$

which is clearly a constant

$$\text{Thus, } (S_1P_1)(S_2P_2) = b^2$$

**21. (d) :** Let the equation of ellipse having centre

$$(1, 2) \text{ be } \frac{(x-1)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$$

$$\text{We have, } ae = 5 \Rightarrow a^2 e^2 = 25$$

$$\text{Also, } \frac{9}{a^2} + \frac{16}{b^2} = 1$$

$$\text{Now, } b^2 = a^2(1 - e^2) = a^2 - 25$$

$$\Rightarrow \frac{9}{a^2} + \frac{16}{a^2 - 25} = 1 \Rightarrow a^4 - 50a^2 + 225 = 0$$

$$\Rightarrow a^2 = 5, 45$$

But  $a^2 = 5$  is clearly rejected

$$\therefore a^2 = 45 \Rightarrow b^2 = 20$$

$$\text{Thus, required equation is } \frac{(x-1)^2}{45} + \frac{(y-2)^2}{20} = 1$$

**22. (d) :** Tangent to the ellipse is

$$y = mx \pm \sqrt{\frac{5m^2}{3} + \frac{5}{2}}$$

$$\text{If it passes through } P(1, 2), \text{ then } (2 - m)^2 = \frac{5m^2}{3} + \frac{5}{2}$$

$$\Rightarrow 4m^2 + 24m - 9 = 0$$

If its roots are  $m_1$  and  $m_2$  then

$$m_1 + m_2 = -6, m_1 m_2 = -\frac{9}{4}$$

$$\Rightarrow |m_1 - m_2| = \sqrt{36 + 9} = 3\sqrt{5}$$

$$\therefore \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{3\sqrt{5}}{1 - \frac{9}{4}} \right| = \frac{12}{\sqrt{5}}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{12}{\sqrt{5}} \right)$$

**23. (c) :** Equation of normal at point 'θ' is

$$\frac{5x}{\cos \theta} - \frac{3y}{\sin \theta} = 16$$

On comparing with  $5x - 3y = 8\sqrt{2}$ , we get

$$\Rightarrow \cos \theta = \sin \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

**24. (c) :** Let the point 'P' be  $(\sqrt{6} \cos \theta, \sqrt{2} \sin \theta)$

A.T.Q.

$$6 \cos^2 \theta + 2 \sin^2 \theta = 4$$

$$\Rightarrow 6 - 4 \sin^2 \theta = 4 \Rightarrow \sin^2 \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4} (\because \theta \text{ lies in first quadrant})$$

**25. (b) :** Equation of tangent at point 'θ' is,

$$\frac{x}{4} \cos \theta + \frac{\sqrt{11}}{16} y \sin \theta = 1$$

It will touch the given circle, if

$$\frac{\left| \frac{\cos \theta}{4} - 1 \right|}{\sqrt{\frac{\cos^2 \theta}{16} + \frac{11 \sin^2 \theta}{256}}} = 4$$

$$\Rightarrow 4 \cos^2 \theta + 8 \cos \theta - 5 = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

**26. (c) :** The given distance is clearly the length of semi major axis

$$\text{Thus, } \sqrt{\frac{a^2 + 2b^2}{2}} = a$$

$$\Rightarrow 2b^2 = a^2 \Rightarrow 2a^2(1 - e^2) = a^2$$

$$\Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$

**27. (a) :** Let the equation of ellipse be  $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

$$\text{We have, } y = -\frac{2a}{\sqrt{1-a^2}}x + \frac{1}{\sqrt{1-a^2}}$$

Comparing it with  $y = mx \pm \sqrt{A^2 m^2 + B^2}$ , we get

$$m = -\frac{2a}{\sqrt{1-a^2}}$$

$$A^2 m^2 + B^2 = \frac{1}{1-a^2}$$

$$\Rightarrow A^2 \cdot \frac{4a^2}{1-a^2} + B^2 = \frac{1}{1-a^2}$$

$$\Rightarrow \frac{a^2(4A^2 - B^2) + B^2}{1-a^2} = \frac{1}{1-a^2} \Rightarrow A^2 = \frac{1}{4}$$

If 'e' be the eccentricity of the ellipse, then

$$A^2 = B^2(1 - e^2)$$

$$\Rightarrow 1 - e^2 = \frac{1}{4} \Rightarrow e = \frac{\sqrt{3}}{2}$$

**28. (d) :** Any normal to the ellipse is

$$\frac{\sqrt{5}x}{\cos \theta} - \frac{2y}{\sin \theta} = 1$$

$$\text{i.e., } y = x \cdot \frac{\sqrt{5}}{2} \tan \theta - \frac{\sin \theta}{2} \quad \dots(i)$$

Since (i) is tangent to  $y^2 = 4x$

$$\therefore y = mx + \frac{1}{m}$$

$$\Rightarrow m = \frac{\sqrt{5}}{2} \tan \theta, \frac{1}{m} = -\frac{\sin \theta}{2}$$

$$\Rightarrow \frac{\sqrt{5}}{4} \cdot \frac{\sin^2 \theta}{\cos \theta} = -1 \Rightarrow \cos \theta = -\frac{1}{\sqrt{5}}, \text{ and } \cos \theta = \sqrt{5}$$

$$\Rightarrow \theta = \pi - \cos^{-1} \left( \frac{1}{\sqrt{5}} \right) \text{ or } \pi + \cos^{-1} \left( \frac{1}{\sqrt{5}} \right)$$

**29. (a) :** Let line AB be  $bx \cos \theta + ay \sin \theta - ab = 0$

Now, combined equation of lines OA and OB is

$$x^2 + y^2 = a^2 \left( \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta \right)^2$$

$$\text{i.e., } x^2 \cdot \sin^2 \theta + y^2 \left( 1 - \frac{a^2}{b^2} \sin^2 \theta \right) - \frac{a}{b} \sin 2\theta \cdot xy = 0$$

Since the lines OA and OB are mutually perpendicular, hence

$$\sin^2 \theta + 1 - \frac{a^2}{b^2} \sin^2 \theta = 0$$

$$\Rightarrow \sin^2 \theta \left( \frac{a^2}{b^2} - 1 \right) = 1 \Rightarrow \sin^2 \theta \left( \frac{1}{1-e^2} - 1 \right) = 1$$

$$\Rightarrow e^2 \cdot \sin^2 \theta = 1 - e^2 \Rightarrow e^2 (1 + \sin^2 \theta) = 1$$

**30. (a) :** Let the extremities of the focal chord be  $P_1(\theta_1)$  and  $P_2(\theta_2)$ .

Equation of chord  $P_1P_2$  is

$$\frac{x}{a} \cos \left( \frac{\theta_1 + \theta_2}{2} \right) + \frac{y}{b} \sin \left( \frac{\theta_1 + \theta_2}{2} \right) = \cos \left( \frac{\theta_1 - \theta_2}{2} \right)$$

It should pass through  $(ae, 0)$  and its slope should be  $\tan \theta$

$$\Rightarrow e \cos \left( \frac{\theta_1 + \theta_2}{2} \right) = \cos \left( \frac{\theta_1 - \theta_2}{2} \right)$$

$$\text{and } -\frac{b}{a} \cot \left( \frac{\theta_1 + \theta_2}{2} \right) = \tan \theta$$

$$\Rightarrow \cot \left( \frac{\theta_1 + \theta_2}{2} \right) = -\frac{a \tan \theta}{b}$$

Now,  $P_1P_2 = P_1S_1 + P_2S_1$

$$= a - ea \cos \theta_1 + a - ea \cos \theta_2$$

$$= 2a - ea(\cos \theta_1 + \cos \theta_2)$$

$$= 2a - 2ae \cos \left( \frac{\theta_1 + \theta_2}{2} \right) \cos \left( \frac{\theta_1 - \theta_2}{2} \right)$$

$$= 2a - 2ae^2 \cos^2 \left( \frac{\theta_1 + \theta_2}{2} \right)$$

$$= 2a - 2ae^2 \cdot \left( \frac{a^2 \tan^2 \theta}{a^2 \tan^2 \theta + b^2} \right)$$

$$= 2a - \frac{2a^3 e^2 \sin^2 \theta}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$$

$$= \frac{2a^3 \sin^2 \theta (1 - e^2) + 2ab^2 \cos^2 \theta}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$$

$$= \frac{2ab^2 \sin^2 \theta + 2ab^2 \cos^2 \theta}{a^2 \sin^2 \theta + b^2 \cos^2 \theta} = \frac{2ab^2}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$$

**31. (a) :** Let the foot of normal is  $(a \cos \theta, b \sin \theta)$ ,

$$\text{then } \frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$$

and the given lines will be identical,

$$\text{Thus, } \frac{m \cos \theta}{a} = \frac{\sin \theta}{b} = \frac{c}{(b^2 - a^2)}$$

$$\Rightarrow \cos \theta = \frac{ac}{m(b^2 - a^2)}, \sin \theta = \frac{bc}{(b^2 - a^2)}$$

$$\Rightarrow \frac{a^2}{m^2} + b^2 = \frac{(b^2 - a^2)^2}{c^2}$$

**32. (c) :** If the point of intersection be  $R(h, k)$ , then

$$h = \frac{a \cos \left( \frac{\theta_1 + \theta_2}{2} \right)}{\cos \left( \frac{\theta_1 - \theta_2}{2} \right)}, k = \frac{b \sin \left( \frac{\theta_1 + \theta_2}{2} \right)}{\cos \left( \frac{\theta_1 - \theta_2}{2} \right)}$$

$$\Rightarrow h = \frac{a \cos \left( \frac{\theta_1 + \theta_2}{2} \right)}{\frac{1}{2}}, k = \frac{b \sin \left( \frac{\theta_1 + \theta_2}{2} \right)}{\frac{1}{2}}$$

$$\text{Thus, } \frac{h^2}{a^2} + \frac{k^2}{b^2} = 4$$

$$\text{Hence, locus is, } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$$

**33. (d) :** Let  $P \equiv (a \cos \theta, b \sin \theta)$

$$\Rightarrow P_1 \equiv (a \cos \theta, 0), P_2 \equiv (0, b \sin \theta)$$

Thus, equation of line  $P_1P_2$  is  $\frac{x}{a \cos \theta} + \frac{y}{b \sin \theta} = 1$

$$\Rightarrow \frac{x/a}{\cos(-\theta)} - \frac{y/b}{\sin(-\theta)} = 1$$

which is clearly a normal to the ellipse of the form

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \text{ where, } \frac{A}{A^2 - B^2} = \frac{\lambda}{a} \text{ and } \frac{B}{A^2 - B^2} = \frac{\lambda}{b}$$

If  $a > b$ , then  $B > A$

Let the eccentricity of the second ellipse be  $e_1$

$$\Rightarrow 1 - e_1^2 = \frac{A^2}{B^2} = \frac{b^2}{a^2} = 1 - e^2 \Rightarrow e_1 = e$$

**34. (a) :** Equation of normal at  $\left( ae, \frac{b^2}{a} \right)$  is

$$\frac{a(x - ae)}{e} = (ay - b^2)$$

It should pass through  $(0, -b)$

$$\Rightarrow \frac{a(0 - ae)}{e} = -a^2 - b^2$$

$$\Rightarrow a^2 = ab + b^2 \Rightarrow \frac{b^2}{a^2} + \frac{b}{a} - 1 = 0$$

$$\Rightarrow \frac{b}{a} = \frac{-1 + \sqrt{5}}{2} \Rightarrow \frac{b^2}{a^2} = \frac{3 - \sqrt{5}}{2} = 1 - e^2$$

$$\Rightarrow e^2 = \frac{\sqrt{5} - 1}{2} \Rightarrow e = \sqrt{\frac{\sqrt{5} - 1}{2}}$$

**35. (c) :** Tangent for  $y^2 = 4x$  is  $y = mx + \frac{1}{m}$

and for  $\frac{x^2}{8} + \frac{y^2}{2} = 1$  is  $y = mx \pm \sqrt{8m^2 + 2}$

For common tangents we must have

$$\frac{1}{m} = \pm \sqrt{8m^2 + 2} \Rightarrow m = \pm \frac{1}{2}$$

Thus, common tangents are

$$y = \frac{x}{2} + 2 \text{ and } y = -\frac{x}{2} - 2$$

i.e.  $2y - x - 4 = 0$  and  $2y + x + 4 = 0$

**36. (c) :** Any normal to the ellipse is

$$\frac{5x}{\cos \theta} - \frac{4y}{\sin \theta} = 9 \text{ i.e., } 5 \sec \theta x - 4 \operatorname{cosec} \theta y - 9 = 0$$

$$\begin{aligned} \therefore L &= \frac{9}{\sqrt{25 \sec^2 \theta + 16 \operatorname{cosec}^2 \theta}} \\ &= \frac{9}{\sqrt{25(1 + \tan^2 \theta) + 16(1 + \cot^2 \theta)}} \\ &= \frac{9}{\sqrt{25 \tan^2 \theta + 16 \cot^2 \theta + 41}} \end{aligned}$$

$$\text{Now, } \frac{25 \tan^2 \theta + 16 \cot^2 \theta}{2} \geq 20$$

$$\Rightarrow 25 \tan^2 \theta + 16 \cot^2 \theta + 41 \geq 81$$

$$\Rightarrow \frac{9}{\sqrt{25 \tan^2 \theta + 16 \cot^2 \theta + 41}} \leq 1 \Rightarrow L \leq 1$$

$\therefore$  Maximum value of  $L$  is 1.

**37. (a) :** Clearly, the tangents have been drawn by taking any point on the director circle of the ellipse. Any point on the director circle can be taken as  $(\sqrt{13} \cos \theta, \sqrt{13} \sin \theta)$ .

Equation of corresponding chord of contact is

$$\frac{\sqrt{13}}{9} \cdot \cos \theta \cdot x + \frac{\sqrt{13}}{4} \sin \theta \cdot y = 1$$

$$\begin{aligned} \therefore \text{Distance from the origin} &= \frac{1}{\sqrt{\frac{13}{81} \cos^2 \theta + \frac{13}{16} \sin^2 \theta}} \\ &= \frac{36}{\sqrt{1053 - 845 \cos^2 \theta}} = \frac{36}{\sqrt{208}} = \frac{9}{\sqrt{13}} \end{aligned}$$

**38. (b) :** Equation of  $AB$  is  $T = 0$ ,

$$\text{i.e. } \frac{5y}{9} = 1 \Rightarrow y = \frac{9}{5}$$

Putting  $y = \frac{9}{5}$  in the equation of ellipse, we get

$$\frac{x^2}{16} + \frac{81}{25 \cdot 9} = 1 \Rightarrow x = \pm \frac{16}{5} \cdot \text{Thus, } AB = \frac{32}{5}$$

$$\text{Distance of } P \text{ from } AB = 5 - \frac{9}{5} = \frac{16}{5}$$

$$\text{Thus, area of } \Delta_{PAB} = \frac{1}{2} \cdot \frac{32}{5} \cdot \frac{16}{5} = \frac{256}{25} \text{ sq. units}$$

**39. (a) :** Other common tangent will clearly be the reflection of  $x - 2y + 4 = 0$  in the  $x$ -axis.

Thus other common tangent will be  $x + 2y + 4 = 0$

**40. (c) :** Let the eccentric angle of point 'P' be  $\theta$ , then equation of tangent and normal at point are respectively

$$\frac{x}{6} \cos \theta + \frac{y}{3} \sin \theta = 1 \text{ and } \frac{6x}{\cos \theta} - \frac{3y}{\sin \theta} = 27$$

$$\text{Thus, } A \equiv \left(0, \frac{3}{\sin \theta}\right) \text{ and } B \equiv \left(\frac{27}{6} \cos \theta, 0\right)$$

$$\text{Area of } \Delta_{OAB} = \frac{1}{2} \cdot \frac{3}{\sin \theta} \cdot \frac{27}{6} \cdot \cos \theta = \frac{27}{4} \text{ (given)}$$

$$\Rightarrow \cot \theta = 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

**41. (c) :** Let  $P \equiv (2 \cos \theta, \sin \theta)$

$$\text{Equation of tangent at 'P' is } \frac{x}{2} \cos \theta + y \sin \theta = 1$$

$$\Rightarrow A \equiv (2 \sec \theta, 0)$$

$$\text{Equation of normal at 'P' is } \frac{2x}{\cos \theta} - \frac{y}{\sin \theta} = 3$$

$$\Rightarrow B \equiv \left(\frac{3}{2} \cos \theta, 0\right)$$

$$\text{Now, } AB = \left| 2 \sec \theta - \frac{3}{2} \cos \theta \right| = 2 \text{ (given)}$$

$$\Rightarrow |4 - 3 \cos^2 \theta| = 4 |\cos \theta| \Rightarrow 3 \cos^2 \theta \pm 4 \cos \theta - 4 = 0$$

$$\Rightarrow \cos \theta = \frac{2}{3}, -\frac{2}{3}$$

**42. (a) :** Equation of chord is

$$\frac{x}{a} \cos \left( \frac{\theta_1 + \theta_2}{2} \right) + \frac{y}{b} \sin \left( \frac{\theta_1 + \theta_2}{2} \right) = \cos \left( \frac{\theta_1 - \theta_2}{2} \right)$$

$$\Rightarrow A \equiv \left( \frac{a \cos \left( \frac{\theta_1 - \theta_2}{2} \right)}{\cos \left( \frac{\theta_1 + \theta_2}{2} \right)}, 0 \right)$$

Since  $OA = c$ , therefore,

$$a \cos \left( \frac{\theta_1 - \theta_2}{2} \right) = c \cdot \cos \left( \frac{\theta_1 + \theta_2}{2} \right)$$

$$\Rightarrow \cos \frac{\theta_1}{2} \cdot \cos \frac{\theta_2}{2} (a - c) = \sin \frac{\theta_1}{2} \cdot \sin \frac{\theta_2}{2} (-c - a)$$

$$\Rightarrow \tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2} = \frac{c - a}{c + a}$$

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**43. (b) :** Equation of tangent at point

$$(\sqrt{18} \cos \theta, \sqrt{32} \sin \theta), \text{ is } \frac{x}{\sqrt{18}} \cos \theta + \frac{y}{\sqrt{32}} \sin \theta = 1$$

$$\text{Its slope} = -\frac{\sqrt{32}}{\sqrt{18}} \cdot \cot \theta = -\frac{4}{3} \quad (\text{given})$$

$$\Rightarrow \cot \theta = 1$$

$$\text{Also, } A \equiv (\sqrt{18} \sec \theta, 0), B \equiv (0, \sqrt{32} \operatorname{cosec} \theta)$$

$$\Rightarrow \Delta_{OAB} = \frac{1}{2} |3\sqrt{2} \sec \theta \cdot 4\sqrt{2} \operatorname{cosec} \theta|$$

$$= 12 \cdot \sqrt{2} \cdot \sqrt{2} = 24 \text{ sq. units}$$

**44. (a) :** Any tangent to  $\frac{x^2}{a^2 + b^2} + \frac{y^2}{b^2} = 1$  is,

$$y = mx \pm \sqrt{(a^2 + b^2)m^2 + b^2}$$

It will also be a tangent to second ellipse if

$$a^2 m^2 + (a^2 + b^2) = (a^2 + b^2) m^2 + b^2$$

$$\Rightarrow b^2 m^2 = a^2 \Rightarrow m = \pm \frac{a}{b}$$

Thus common tangents are

$$y = \pm \frac{a}{b} x \pm \sqrt{(a^2 + b^2) \cdot \frac{a^2}{b^2} + b^2}$$

$$\text{i.e., } by = \pm ax \pm \sqrt{a^4 + b^4 + a^2 b^2}$$

**45. (a) :** Let the extremities of the chord be

$$P_1 \equiv (a \cos \theta, b \sin \theta) \text{ and } P_2 \equiv (-a \sin \theta, b \cos \theta)$$

$$\text{Now, } P_1 P_2^2 = a^2 (\cos \theta + \sin \theta)^2 + b^2 (\sin \theta - \cos \theta)^2$$

$$= a^2 + b^2 + (a^2 - b^2) 2 \sin \theta \cos \theta$$

$$\leq a^2 + b^2 + a^2 - b^2 = 2a^2$$

$$\Rightarrow P_1 P_2 \leq a\sqrt{2}$$

**46. (d) :** Let the ellipse be  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

If the line  $y = mx + c$  be its tangent, then

$$c^2 = a^2 m^2 + b^2$$

$$\text{Tangent is given to be } x + 4y - 10 = 0$$

$$\Rightarrow \frac{m}{1} = -\frac{1}{4} = \frac{c}{-10} \Rightarrow m = -\frac{1}{4}, c = \frac{5}{2} \therefore \frac{25}{4} = \frac{a^2}{16} + b^2$$

$$\Rightarrow a^2 + 16b^2 = 100$$

We also have  $P(4, -1)$  to be a point on the ellipse,

$$\therefore \frac{16}{a^2} + \frac{1}{b^2} = 1$$

$$\Rightarrow a^2 + 16b^2 = a^2 b^2 \Rightarrow a^2 b^2 = 100 \Rightarrow ab = 10$$

$$\therefore \text{Area of ellipse} = \pi ab = 10\pi \text{ sq. units}$$

**47. (b) :** If 'O' be the center of ellipse then for triangle  $S_1 S_2 B$  to be right angled, we must have

$$\angle O B S_1 = \angle O B S_2 = \frac{\pi}{4}.$$

$$\text{Thus, } OS_1 = OB = OS_2 \Rightarrow ae = b \Rightarrow a^2 e^2 = a^2 - a^2 e^2$$

$$\Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$

**48. (c) :** Let  $P \equiv (h, k)$

$$\text{Equation of chord of contact is } \frac{xh}{a^2} + \frac{yk}{b^2} = 1$$

$$\text{i.e., } xb^2 h + ya^2 k - a^2 b^2 = 0$$

It will touch the circle  $x^2 + y^2 = c^2$ , if

$$\frac{a^2 b^2}{\sqrt{b^4 h^2 + a^4 k^2}} = c$$

Hence, locus of point 'P' is

$$b^4 x^2 + a^4 y^2 = \frac{a^4 b^4}{c^2} \text{ i.e., } \frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$$

**49. (c) :** Slope of tangent must be  $-\frac{9}{8}$ .

Slope of tangent at any point  $P(\theta)$  on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } -\frac{b}{a} \cot \theta$$

$$\text{Thus, } -\frac{9}{8} = -\frac{2}{3} \cot \theta \Rightarrow \tan \theta = \frac{16}{27}$$

$$\Rightarrow \cos \theta = -\frac{27}{\sqrt{985}}, \sin \theta = \frac{16}{\sqrt{985}}$$

$$\text{or } \cos \theta = -\frac{27}{\sqrt{985}}, \sin \theta = -\frac{16}{\sqrt{985}}$$

Thus, required point is

$$\left( \frac{27}{2\sqrt{985}}, \frac{16}{3\sqrt{985}} \right) \text{ or } \left( -\frac{27}{2\sqrt{985}}, -\frac{16}{3\sqrt{985}} \right)$$

**50. (b) :** Let  $P \equiv (h, k)$

Any tangent to the ellipse is

$$y = mx \pm \sqrt{16m^2 + 9}$$

If it passes through 'P', then

$$(k - mh)^2 = 16m^2 + 9$$

$$\Rightarrow m^2(h^2 - 16) - 2mhk + (k^2 - 9) = 0$$

If its roots are  $m_1$  and  $m_2$ , then

$$m_1 m_2 = \frac{k^2 - 9}{h^2 - 16}$$

Since, the drawn tangents meet the coordinate axes at concyclic points, thus,

$$m_1 m_2 = 1 \Rightarrow k^2 - 9 = h^2 - 16$$

Hence required locus is,

$$x^2 - y^2 = 7.$$



# KEY CONCEPTS

## ON Conic Sections

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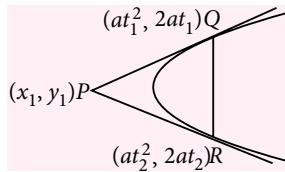
### IMPORTANT FACTS AND FORMULAE

- The area of the triangle inscribed in the parabola  $y^2 = 4ax$  is  $\frac{1}{8a} |(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ , where  $y_1, y_2, y_3$  are the ordinates of the vertices.
- The length of the side of an equilateral triangle inscribed in the parabola  $y^2 = 4ax$  is  $8a\sqrt{3}$ . (one angular point is at the vertex).
- $y^2 = 4a(x + a)$  is the equation of the parabola whose focus is the origin and the axis is  $x$ -axis.
- $y^2 = 4a(x - a)$  is the equation of parabola whose axis is  $x$ -axis and  $y$ -axis is directrix.
- $x^2 = 4a(y + a)$  is the equation of parabola whose focus is the origin and the axis is  $y$ -axis.
- $x^2 = 4a(y - a)$  is the equation of parabola whose axis is  $y$ -axis and the  $x$ -axis is directrix.
- The equation of the parabola whose vertex and focus are on  $x$ -axis at a distance  $a$  and  $a'$  respectively from the origin is  $y^2 = 4(a' - a)(x - a)$ .
- The equation of the parabola whose axis is parallel to  $x$ -axis is  $x = Ay^2 + By + C$  and  $y = Ax^2 + Bx + C$  is a parabola with its axis parallel to  $y$ -axis.
- If the straight line  $lx + my + n = 0$  touches the parabola  $y^2 = 4ax$ , then  $ln = am^2$ .
- If the line  $x\cos\alpha + y\sin\alpha = p$  touches the parabola  $y^2 = 4ax$  then  $p\cos\alpha + a\sin^2\alpha = 0$  and point of contact is  $(a\tan^2\alpha, -2a\tan\alpha)$ .
- If the line  $\frac{x}{l} + \frac{y}{m} = 1$  touches the parabola  $y^2 = 4a(x + b)$ , then  $m^2(l + b) + al^2 = 0$ .
- If the two parabolas  $y^2 = 4x$  and  $x^2 = 4y$  intersect at point  $P$ , whose abscissae is not zero, then the tangent to each curve at  $P$ , make complementary angle with the  $x$ -axis.
- Tangents at the extremities of any focal chord of a parabola meet at right angle on the directrix.
- Area of the triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.
- If the tangents at the points  $P$  and  $Q$  on a parabola meet in  $T$ , then  $ST$  is the geometric mean between  $SP$  and  $SQ$ , i.e.,  $ST^2 = SP \cdot SQ$
- Tangent at one extremity of the focal chord of a parabola is parallel to the normal at the other extremity.
- The angle of intersection of two parabolas  $y^2 = 4ax$  and  $x^2 = 4by$  is given by  $\tan^{-1} \left| \frac{3a^{1/3}b^{1/3}}{2(a^{2/3} + b^{2/3})} \right|$ .
- The equation of the common tangent to the parabolas  $y^2 = 4ax$  and  $x^2 = 4by$  is  $\frac{1}{a^3}x + \frac{1}{b^3}y + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$ .
- The line  $lx + my + n = 0$  is a normal to the parabola  $y^2 = 4ax$ , if  $a(l^2 + 2m^2) + m^2n = 0$ .
- If the normals at points  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  on the parabola  $y^2 = 4ax$  meet on the parabola, then  $t_1t_2 = 2$ .
- If the normal at a point  $P(at^2, 2at)$  to the parabola  $y^2 = 4ax$  subtends a right angle at the vertex of the parabola then  $t^2 = 2$ .
- If the normal to a parabola  $y^2 = 4ax$ , makes an angle  $\phi$  with the axis, then it will cut the curve again at an angle  $\tan^{-1} \left( \frac{1}{2} \tan \phi \right)$ .
- If the normal at two points  $P$  and  $Q$  of a parabola  $y^2 = 4ax$  intersect at a third point  $R$  on the curve. Then the product of the ordinate of  $P$  and  $Q$  is  $8a^2$ .

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He trains IIT and Olympiad aspirants.

- The chord of contact joining the point of contact of two perpendicular tangents always passes through focus.

- If tangents are drawn from the point  $(x_1, y_1)$  to the parabola  $y^2 = 4ax$ , then the length of their chord of contact is



$$\frac{1}{|a|} \sqrt{(y_1^2 - 4ax_1)(y_1^2 + 4a^2)}$$

- The area of the triangle formed by the tangents drawn from  $(x_1, y_1)$  to  $y^2 = 4ax$  and their chord of contact is  $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$ .

- If one extremity of a focal chord is  $(at_1^2, 2at_1)$ , then the other extremity  $(at_2^2, 2at_2)$  becomes  $\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$  by virtue of relation  $t_1 t_2 = -1$  and length of chord  $= a \left(t + \frac{1}{t}\right)^2$ .

- The length of the chord joining two points ' $t_1$ ' and ' $t_2$ ' on the parabola  $y^2 = 4ax$  is  $a(t_1 - t_2)\sqrt{(t_1 + t_2)^2 + 4}$

- The length of intercept made by line  $y = mx + c$  between the parabola  $y^2 = 4ax$  is  $\frac{4}{m^2} \sqrt{a(1 + m^2)(a - mc)}$ .

- Locus of mid-points of all chords which is inscribed a right angle on the vertices of parabola is parabola.

- The focal chord of parabola  $y^2 = 4ax$  making an angle  $\alpha$  with the  $x$ -axis is of length  $4a \operatorname{cosec}^2 \alpha$  and perpendicular on it from the vertex is  $asin \alpha$ .

- The length of a focal chord of a parabola varies inversely as the square of its distance from the vertex.

- If  $l_1$  and  $l_2$  are the length of segments of a focal chord of a parabola, then its latus-rectum is  $\frac{4l_1 l_2}{l_1 + l_2}$ .

- The semi-latus rectum of the parabola  $y^2 = 4ax$  is the harmonic mean between the segments of any focal chord of the parabola.

- The straight line  $lx + my + n = 0$  touches the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , if  $a^2 l^2 + b^2 m^2 = n^2$ .

- The line  $x \cos \alpha + y \sin \alpha = p$  touches the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , if  $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = p^2$  and that

$$\text{point of contact is } \left( \frac{a^2 \cos \alpha}{p}, \frac{b^2 \sin \alpha}{p} \right).$$

- A circle of radius  $r$  is concentric with the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then the common tangent is inclined

$$\text{to the major axis at an angle } \tan^{-1} \sqrt{\frac{r^2 - b^2}{a^2 - r^2}}.$$

- The locus of the foot of the perpendicular drawn from centre upon any tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $(x^2 + y^2)^2 = a^2 x^2 + b^2 y^2$  or  $r^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$ . (In terms of polar coordinates)

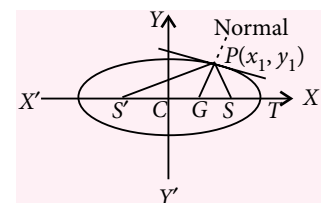
- The locus of the mid points of the portion of the tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  intercepted between the axes is  $a^2 y^2 + b^2 x^2 = 4x^2 y^2$ .

- If  $y = mx + c$  is the normal to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then condition of normality is  $c^2 = \frac{m^2(a^2 - b^2)^2}{(a^2 + b^2 m^2)}$ .

- The straight line  $lx + my + n = 0$  is a normal to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , if  $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$ .

- Four normals can be drawn from a point to an ellipse.

- If  $S$  be the focus and  $G$  be the point where the normal at  $P$  meets the axis of an ellipse, then  $SG = e$ ,  $SP$ , and the



- tangent and normal at  $P$  bisect the external and internal angles between the focal distances of  $P$ .

- Any point  $P$  of an ellipse is joined to the extremities of the major axis then the portion of a directrix intercepted by them subtends a right angle at the corresponding focus.

- The equations to the normals at the end of the latus rectum and that each passes through an end of the minor axis, if  $e^4 + e^2 - 1 = 0$ .

- The area of the triangle formed by the three points, on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , whose eccentric angles are  $\theta$ ,  $\phi$  and  $\psi$  is  $2ab \sin\left(\frac{\phi - \psi}{2}\right) \sin\left(\frac{\psi - \theta}{2}\right) \sin\left(\frac{\theta - \phi}{2}\right)$
- If the point of intersection of the ellipses  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and  $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$  be at the extremities of the conjugate diameters of the former, then  $\frac{a^2}{\alpha^2} + \frac{b^2}{\beta^2} = 2$ .
- The sum of the squares of the reciprocal of two perpendicular diameters of an ellipse is constant.
- In an ellipse, the major axis bisects all chords parallel to the minor axis and vice-versa, therefore major and minor axes of an ellipse are conjugate diameters of the ellipse but they do not satisfy the condition  $m_1 \cdot m_2 = -b^2/a^2$  and are the only perpendicular conjugate diameters.
- The foci of a hyperbola and its conjugate are concyclic.
- Two tangents can be drawn from an outside point to a hyperbola.
- If the straight line  $lx + my + n = 0$  touches the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then  $a^2l^2 - b^2m^2 = n^2$ .
- If the straight line  $x \cos \alpha + y \sin \alpha = p$  touches the hyperbolas  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then  $a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$ .
- If the line  $lx + my + n = 0$  will be normal to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then  $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2 + b^2)^2}{n^2}$
- In general, four normals can be drawn to a hyperbola from any point.
- If  $\alpha, \beta, \gamma$  are the eccentric angles of three points on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , the normals at which are concurrent, then  $\sin(\alpha + \beta) + \sin(\beta + \gamma) + \sin(\gamma + \alpha) = 0$
- The feet of the normals to  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  from  $(h, k)$  lie on  $a^2y(x - h) + b^2x(y - k) = 0$ .
- The length of chord cut off by hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  from the line  $y = mx + c$  is  $\frac{2ab\sqrt{[c^2 - (a^2m^2 - b^2)](1 + m^2)}}{(b^2 - a^2m^2)}$ .
- If the chord joining two points  $(a \sec \theta_1, b \tan \theta_1)$   $(a \sec \theta_2, b \tan \theta_2)$  passes through the focus of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then  $\tan \frac{\theta_1}{2} \tan \frac{\theta_2}{2} = \frac{1 - e}{1 + e}$ .
- If the polars of  $(x_1, y_1)$  and  $(x_2, y_2)$  with respect to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  are at right angles, then  $\frac{x_1x_2}{y_1y_2} + \frac{a^4}{b^4} = 0$ .
- The parallelogram formed by the tangents at the extremities of conjugate diameters of a hyperbola has its vertices lying on the asymptotes and is of constant area.
- The product of length of perpendiculars drawn from any point on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  to the asymptotes is  $\frac{a^2b^2}{a^2 + b^2}$ .
- If the normal at  $\left(ct, \frac{c}{t}\right)$  on the curve  $xy = c^2$  meets the curve again in 't', then  $t' = -\frac{1}{t^3}$ .
- A triangle has its vertices on a rectangular hyperbola; then the orthocentre of the triangle also lies on the same hyperbola.
- All conics passing through the intersection of two rectangular hyperbolas are themselves rectangular hyperbolas.
- An infinite number of triangles can be inscribed in the rectangular hyperbola  $xy = c^2$  whose all sides touch the parabola  $y^2 = 4ax$ .

## PROBLEMS

### Single Correct Answer Type

1. The equation of the common tangent of the parabolas  $x^2 = 108y$  and  $y^2 = 32x$ , is

- (a)  $2x + 3y = 36$  (b)  $2x + 3y + 36 = 0$   
(c)  $3x + 2y = 36$  (d)  $3x + 2y + 36 = 0$

2. The line  $x \cos \alpha + y \sin \alpha = p$  will touch the parabola  $y^2 = 4a(x + a)$ , if

- (a)  $p \cos \alpha + a = 0$  (b)  $p \cos \alpha - a = 0$   
(c)  $a \cos \alpha + p = 0$  (d)  $a \cos \alpha - p = 0$

3. The angle of intersection between the curves  $y^2 = 4x$  and  $x^2 = 32y$  at point  $(16, 8)$ , is

- (a)  $\tan^{-1}\left(\frac{3}{5}\right)$  (b)  $\tan^{-1}\left(\frac{4}{5}\right)$   
(c)  $\pi$  (d)  $\frac{\pi}{2}$

4. If  $y_1, y_2$  are the ordinates of two points  $P$  and  $Q$  on the parabola and  $y_3$  is the ordinate of the point of intersection of tangents at  $P$  and  $Q$ , then

- (a)  $y_1, y_2, y_3$  are in A.P. (b)  $y_1, y_3, y_2$  are in A.P.  
(c)  $y_1, y_2, y_3$  are in G.P. (d)  $y_1, y_3, y_2$  are in G.P.

5. The equation of the common tangent touching the circle  $(x - 3)^2 + y^2 = 9$  and the parabola  $y^2 = 4x$  above the  $x$ -axis, is

- (a)  $\sqrt{3}y = 3x + 1$  (b)  $\sqrt{3}y = -(x + 3)$   
(c)  $\sqrt{3}y = x + 3$  (d)  $\sqrt{3}y = -(3x + 1)$

6. The angle between the tangents drawn from the points  $(1, 4)$  to the parabola  $y^2 = 4x$  is

- (a)  $\frac{\pi}{2}$  (b)  $\frac{\pi}{3}$  (c)  $\frac{\pi}{4}$  (d)  $\frac{\pi}{6}$

7. Let a circle tangent to the directrix of a parabola  $y^2 = 2ax$  has its centre coinciding with the focus of the parabola. Then the point of intersection of the parabola and circle is

- (a)  $(a, -a)$  (b)  $(a/2, a/2)$   
(c)  $(a/2, \pm a)$  (d)  $(\pm a, a/2)$

8. An ellipse passes through the point  $(-3, 1)$  and its eccentricity is  $\frac{\sqrt{2}}{5}$ . The equation of the ellipse is

- (a)  $3x^2 + 5y^2 = 32$  (b)  $3x^2 + 5y^2 = 25$   
(c)  $3x^2 + y^2 = 4$  (d)  $3x^2 + y^2 = 9$

9. The locus of the point of intersection of perpendicular tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , is

- (a)  $x^2 + y^2 = a^2 - b^2$  (b)  $x^2 - y^2 = a^2 - b^2$   
(c)  $x^2 + y^2 = a^2 + b^2$  (d)  $x^2 - y^2 = a^2 + b^2$

10. If the eccentricity of the two ellipse  $\frac{x^2}{169} + \frac{y^2}{25} = 1$  and  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  are equal, then the value of  $a/b$  is

- (a)  $5/13$  (b)  $6/13$  (c)  $13/5$  (d)  $13/6$

11. The eccentricity of the curve represented by the equation  $x^2 + 2y^2 - 2x + 3y + 2 = 0$  is

- (a)  $0$  (b)  $1/2$  (c)  $1/\sqrt{2}$  (d)  $\sqrt{2}$

12. The equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13, is

- (a)  $25x^2 - 144y^2 = 900$  (b)  $144x^2 - 25y^2 = 900$   
(c)  $144x^2 + 25y^2 = 900$  (d)  $25x^2 + 144y^2 = 900$

13. The vertices of a hyperbola are at  $(0, 0)$  and  $(10, 0)$  and one of its foci is at  $(18, 0)$ . The equation of the hyperbola is

- (a)  $\frac{x^2}{25} - \frac{y^2}{144} = 1$  (b)  $\frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$   
(c)  $\frac{x^2}{25} - \frac{(y-5)^2}{144} = 1$  (d)  $\frac{(x-5)^2}{25} - \frac{(y-5)^2}{144} = 1$

14. The equation of the hyperbola whose foci are  $(6, 4)$  and  $(-4, 4)$  and eccentricity 2 is given by

- (a)  $12x^2 - 4y^2 - 24x + 32y - 127 = 0$   
(b)  $12x^2 + 4y^2 + 24x - 32y - 127 = 0$   
(c)  $12x^2 - 4y^2 - 24x - 32y + 127 = 0$   
(d)  $12x^2 - 4y^2 + 24x + 32y + 127 = 0$

15. The latus rectum of the hyperbola  $9x^2 - 16y^2 + 72x - 32y - 16 = 0$  is

- (a)  $\frac{9}{2}$  (b)  $-\frac{9}{2}$  (c)  $\frac{32}{3}$  (d)  $-\frac{32}{3}$

16. If  $m_1$  and  $m_2$  are the slopes of the tangents to the hyperbola  $\frac{x^2}{25} - \frac{y^2}{16} = 1$  which pass through the point  $(6, 2)$ , then

- (a)  $m_1 + m_2 = \frac{24}{11}$  (b)  $m_1 m_2 = \frac{10}{11}$   
(c)  $m_1 + m_2 = \frac{48}{11}$  (d)  $m_1 m_2 = \frac{11}{20}$

17. Let  $E$  be the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and  $C$  be the circle  $x^2 + y^2 = 9$ . Let  $P$  and  $Q$  be the points  $(1, 2)$  and  $(2, 1)$

respectively. Then

- (a) Q lies inside C but outside E
- (b) Q lies outside both C and E
- (c) P lies inside both C and E
- (d) P lies inside C but outside E

18. The value of  $m$ , for which the line  $y = mx + \frac{25\sqrt{3}}{3}$ ,

is a normal to the conic  $\frac{x^2}{16} - \frac{y^2}{9} = 1$ , is

- (a)  $\sqrt{3}$  (b)  $-\frac{2}{\sqrt{3}}$  (c)  $-\frac{\sqrt{3}}{2}$  (d) 1

19. If  $\theta$  is the acute angle of intersection at a real point of intersection of the circle  $x^2 + y^2 = 5$  and the parabola  $y^2 = 4x$  then  $\tan\theta$  is equal to

- (a) 1 (b)  $\sqrt{3}$  (c) 3 (d)  $\frac{1}{\sqrt{3}}$

20. The equation of the hyperbola in the standard form (with transverse axis along the  $x$ -axis) having the length of the latus rectum = 9 units and eccentricity =  $5/4$  is

- (a)  $\frac{x^2}{16} - \frac{y^2}{18} = 1$  (b)  $\frac{x^2}{36} - \frac{y^2}{27} = 1$
- (c)  $\frac{x^2}{64} - \frac{y^2}{36} = 1$  (d)  $\frac{x^2}{36} - \frac{y^2}{64} = 1$
- (e)  $\frac{x^2}{16} - \frac{y^2}{9} = 1$

21. The equations of the common tangents of the curves  $x^2 + 4y^2 = 8$  and  $y^2 = 4x$  are

- (a)  $x + 2y + 4 = 0$  (b)  $x - 2y + 4 = 0$
- (c)  $2x + y = 4$  (d)  $2x - y + 4 = 0$

22. A straight line touches the rectangular hyperbola  $9x^2 - 9y^2 = 8$  and the parabola  $y^2 = 32x$ . An equation of the line is

- (a)  $9x + 3y - 8 = 0$  (b)  $9x - 3y + 8 = 0$
- (c)  $9x + 3y + 8 = 0$  (d)  $9x - 3y - 8 = 0$

23. A hyperbola having the transverse axis of length  $\frac{1}{2}$  unit is confocal with the ellipse  $3x^2 + 4y^2 = 12$ , then

- (a) Equation of the hyperbola is  $\frac{x^2}{15} - \frac{y^2}{1} = \frac{1}{16}$
- (b) Eccentricity of the hyperbola is 4
- (c) Distance between the directrices of the hyperbola is  $\frac{1}{8}$  units
- (d) Length of latus rectum of the hyperbola is  $\frac{15}{2}$  units

## Comprehension Type

### Paragraph for Q. No. 24 to 26

Consider the circles  $x^2 + y^2 = a^2$  and  $x^2 + y^2 = b^2$  where  $b > a > 0$ . Let  $A(-a, 0)$ ,  $B(a, 0)$ . A parabola passes through the points  $A, B$  and its directrix is a tangent to  $x^2 + y^2 = b^2$ . If the locus of focus of the parabola is a conic then

24. The eccentricity of the conic is

- (a)  $2a/b$  (b)  $b/a$  (c)  $a/b$  (d) 1

25. The foci of the conic are

- (a)  $(\pm 2a, 0)$  (b)  $(0, \pm a)$
- (c)  $(\pm a, 2a)$  (d)  $(\pm a, 0)$

26. Area of triangle formed by a latusrectum and the lines joining the end points of the latusrectum and the centre of the conic is

- (a)  $\frac{a}{b}(b^2 - a^2)$  (b)  $2ab$
- (c)  $ab/2$  (d)  $4ab/3$

### Paragraph for Q. No. 27 to 29

$P$  is any point of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .  $S$  and  $S'$  are

foci and  $e$  is the eccentricity of ellipse.  $\angle PSS' = \alpha$  and  $\angle PS'S = \beta$

27.  $\tan \frac{\alpha}{2} \tan \frac{\beta}{2}$  is equal to

- (a)  $\frac{2e}{1-e}$  (b)  $\frac{1+e}{1-e}$
- (c)  $\frac{1-e}{1+e}$  (d)  $\frac{2e}{1+e}$

28. Locus of incentre of triangle  $PSS'$  is

- (a) an ellipse (b) hyperbola
- (c) parabola (c) circle

29. Eccentricity of conic, which is locus of incentre of triangle  $PSS'$

- (a)  $\sqrt{\frac{e}{1+e}}$  (b)  $\sqrt{\frac{2e}{1+e}}$
- (c)  $\sqrt{\frac{2e}{1-e}}$  (d)  $\sqrt{\frac{e}{1-e}}$

### Paragraph for Q. No. 30 to 32

A point  $P$  moves such that sum of the slopes of the normals drawn from it to the hyperbola  $xy = 16$  is equal to the sum of ordinates of feet of normals. The locus of  $P$  is a curve  $C$ .

30. The equation of the curve C is

- (a)  $x^2 = 4y$  (b)  $x^2 = 16y$   
(c)  $x^2 = 12y$  (d)  $y^2 = 8x$

31. If the tangent to the curve C cuts the coordinate axis in A and B, then the locus of the middle point of AB is

- (a)  $x^2 = 4y$  (b)  $x^2 = 2y$   
(c)  $x^2 + 2y = 0$  (d)  $x^2 + 4y = 0$

32. Area of the equilateral triangle inscribed in a curve C having one vertex is the vertex of curve C.

- (a)  $772\sqrt{3}$  sq. units (b)  $776\sqrt{3}$  sq. units  
(c)  $760\sqrt{3}$  sq. units (d)  $768\sqrt{3}$  sq. units

### Matrix – Match Type

33. Match the following

Consider the parabola  $y^2 = 12x$

Column I		Column II	
A.	Tangent and normal at the extremities of the latus rectum intersect the x-axis at T & G respectively. The coordinates of middle point of T & G are	P.	(0, 0)
B.	Variable chords of the parabola passing through a fixed point K on the axis, such that sum of the reciprocals of two parts of the chord through K, is a constant. Coordinates of K are	Q.	(3, 0)
C.	AB and CD are the chords of a parabola which intersect at a point E on the axis. The radical axis of the two circles described on AB and CD as diameter always passes through the point	R.	(12, 0)

### Integer Answer Type

34. A line passing through (21, 30) and normal to the curve  $y = 2\sqrt{x}$ . If m is slope of the normal then  $m + 6 =$

35. A circle concentric to an ellipse  $\frac{4x^2}{289} + \frac{4y^2}{\lambda^2} = 1$  ( $\lambda < \frac{17}{2}$ ) passes through foci  $F_1$  and  $F_2$  cuts the ellipse at 'P' such that area of triangle  $PF_1F_2$  is 30 sq. units. If  $F_1F_2 = 13K$  where  $K \in \mathbb{Z}$  then  $K =$

36. The equation of the curve on reflection of the ellipse  $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$  about the line  $x - y - 2 = 0$

is  $16x^2 + 9y^2 + ax - 36y + b = 0$  then the value of  $a + b - 125 =$

37. The area of the quadrilateral formed by the tangents at the end point of latus rectum to the ellipse  $\frac{x^2}{9} + \frac{y^2}{5} = 1$  is  $3K$ . Then K is equal to

38. The equation of Asymptotes of  $xy + 2x + 4y + 6 = 0$  is  $xy + 2x + 4y + C = 0$ , then  $C =$

39. If e is the eccentricity of the hyperbola  $(5x - 10)^2 + (5y + 15)^2 = (12x - 5y + 1)^2$  then  $\frac{25e}{13}$  is equal to

40. If the angle between the asymptotes of hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is  $\frac{\pi}{3}$ . Then the eccentricity of conjugate hyperbola is

### SOLUTIONS

1. (b):  $S_1 \equiv x^2 - 108y = 0$

$$T \equiv xx_1 - 2a(y + y_1) = 0 \Rightarrow xx_1 - 54 \left( y + \frac{x_1^2}{108} \right) = 0$$

$$S_2 \equiv y^2 - 32x = 0$$

$$T \equiv yy_2 - 2a(x + x_2) = 0 \Rightarrow yy_2 - 16 \left( x + \frac{y_2^2}{32} \right) = 0$$

$$\therefore \frac{x_1}{16} = \frac{54}{y_2} = \frac{-x_1^2}{y_2^2} = r \Rightarrow x_1 = 16r \text{ and } y_2 = \frac{54}{r}$$

$$\therefore \frac{-(16r)^2}{(54/r)^2} = r \Rightarrow r = -\frac{9}{4}$$

$$x_1 = -36, y_2 = -24, y_1 = \frac{(-36)^2}{108} = 12, x_2 = \frac{(-24)^2}{32} = 18$$

$\therefore$  Equation of common tangent is

$$(y - 12) = \frac{-36}{54}(x + 36) \Rightarrow 2x + 3y + 36 = 0$$

2. (a):  $x \cos \alpha + y \sin \alpha - p = 0$  ... (i)

$$2ax - yy_1 + 2a(x_1 + 2a) = 0 \quad \dots (ii)$$

$$\text{From (i) and (ii), } \frac{\cos \alpha}{2a} = \frac{\sin \alpha}{-y_1} = \frac{-p}{2a(x_1 + 2a)}$$

$$\Rightarrow y_1 = -2a \tan \alpha \text{ and } x_1 = -p \sec \alpha - 2a$$

$$\therefore y^2 = 4a(x + a) \Rightarrow 4a^2 \tan^2 \alpha = -4a(p \sec \alpha + a)$$

$$\Rightarrow p \cos \alpha + a = 0$$

3. (a) : Using formula

$$\theta = \tan^{-1} \frac{3a^{1/3}b^{1/3}}{2(a^{2/3} + b^{2/3})}, \text{ where } a=1 \text{ and } b=8$$

$$\therefore \theta = \tan^{-1} \frac{6}{2(1+4)} = \tan^{-1} \left( \frac{3}{5} \right)$$

4. (b) : Let the coordinates of P and Q be  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  respectively. Then  $y_1 = 2at_1$  and  $y_2 = 2at_2$ . The coordinates of the point of intersection of the tangents at P and Q are  $\{at_1t_2, a(t_1 + t_2)\}$

$$\therefore y_3 = a(t_1 + t_2)$$

$$\Rightarrow y_3 = \frac{y_1 + y_2}{2} \Rightarrow y_1, y_3 \text{ and } y_2 \text{ are in A.P.}$$

5. (c) : Any tangent to  $y^2 = 4x$  is  $y = mx + \frac{1}{m}$ . It touches

$$\text{the circle, if } 3 = \left| \frac{3m + \frac{1}{m}}{\sqrt{1+m^2}} \right|$$

$$\Rightarrow 9(1+m^2) = \left( 3m + \frac{1}{m} \right)^2$$

$$\Rightarrow \frac{1}{m^2} = 3$$

$$\therefore m = \pm \frac{1}{\sqrt{3}}$$

For the common tangent to be above the x-axis,

$$m = \frac{1}{\sqrt{3}}$$

$$\therefore \text{Common tangent is, } y = \frac{1}{\sqrt{3}}x + \sqrt{3} \Rightarrow \sqrt{3}y = x + 3$$

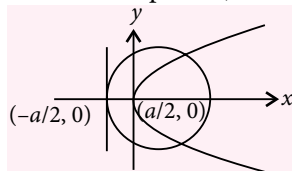
6. (b)

7. (c) : Given parabola is  $y^2 = 2ax$

$\therefore$  Focus  $(a/2, 0)$  and directrix is given by  $x = -a/2$ , as circle touches the directrix.

$\therefore$  Radius of circle = distance from the point  $(a/2, 0)$  to the line  $x = -a/2$

$$= \sqrt{\left( \frac{a}{2} + \frac{a}{2} \right)^2} = a$$



$$\therefore \text{Equation of circle be } \left( x - \frac{a}{2} \right)^2 + y^2 = a^2 \quad \dots(i)$$

$$\text{Also } y^2 = 2ax \quad \dots(ii)$$

$$\text{Solving (i) and (ii) we get, } x = \frac{a}{2}, -\frac{3a}{2}$$

Putting these values in  $y^2 = 2ax$ , we get,

$$y = \pm a \text{ at } x = \frac{a}{2}$$

$$\text{and } y = a\sqrt{-3} \text{ at } x = -\frac{3a}{2}$$

[Which is imaginary value of y]

$\therefore$  Required points are  $(a/2, \pm a)$

8. (a) : Let the equation of ellipse be  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   
 $\therefore$  It passes through  $(-3, 1)$

$$\text{So, } \frac{9}{a^2} + \frac{1}{b^2} = 1 \Rightarrow 9 + \frac{a^2}{b^2} = a^2 \quad \dots(i)$$

Given eccentricity is  $\sqrt{2/5}$

$$\text{So, } \frac{2}{5} = 1 - \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{3}{5} \quad \dots(ii)$$

$$\text{From equation (i) and (ii), } a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$$

Hence, required equation of ellipse is  $3x^2 + 5y^2 = 32$

9. (c) : Let point be  $(h, k)$ . Their pair of tangent will be

$$\left( \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right) \left( \frac{h^2}{a^2} + \frac{k^2}{b^2} - 1 \right) = \left( \frac{hx}{a^2} + \frac{yk}{b^2} - 1 \right)^2$$

Pair of tangents will be perpendicular, if

coefficient of  $x^2$  + coefficient of  $y^2 = 0$

$$\Rightarrow \frac{k^2}{a^2b^2} + \frac{h^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2} \Rightarrow h^2 + k^2 = a^2 + b^2$$

$$\text{Replace } (h, k) \text{ by } (x, y) \Rightarrow x^2 + y^2 = a^2 + b^2$$

10. (c) : In the first case,  $e = \sqrt{1 - (25/169)}$

In the second case,  $e' = \sqrt{1 - (b^2/a^2)}$

According to the given condition,

$$\sqrt{1 - b^2/a^2} = \sqrt{1 - (25/169)}$$

$$\Rightarrow b/a = 5/13 \quad (\because a > 0, b > 0) \Rightarrow a/b = 13/5$$

11. (c) : Equation  $x^2 + 2y^2 - 2x + 3y + 2 = 0$  can be written as

$$\frac{(x-1)^2}{2} + \left( y + \frac{3}{4} \right)^2 = \frac{1}{16} \Rightarrow \frac{(x-1)^2}{(1/8)} + \frac{\left( y + \frac{3}{4} \right)^2}{(1/16)} = 1,$$

which is an ellipse with  $a^2 = \frac{1}{8}$  and  $b^2 = \frac{1}{16}$

$$\therefore \frac{1}{16} = \frac{1}{8}(1 - e^2) \Rightarrow \frac{1}{16} = \frac{1}{8}(1 - e^2) \Rightarrow e = \frac{1}{\sqrt{2}}$$

**12. (a) :** Conjugate axis is 5 and distance between foci = 13  $\Rightarrow 2b = 5$  and  $2ae = 13$

Now, also we know for hyperbola

$$b^2 = a^2(e^2 - 1) \Rightarrow \frac{25}{4} = \frac{(13)^2}{4e^2}(e^2 - 1)$$

$$\Rightarrow \frac{25}{4} = \frac{169}{4} - \frac{169}{4e^2} \text{ or } e^2 = \frac{169}{144} \Rightarrow e = \frac{13}{12}$$

$$\Rightarrow a = 6, b = \frac{5}{2}$$

$\therefore$  Required equation is  $25x^2 - 144y^2 = 900$

**13. (b) :**  $2a = 10, \Rightarrow a = 5$

$$ae - a = 8 \text{ or } e = 1 + \frac{8}{5} = \frac{13}{5}$$

$$\therefore b = 5\sqrt{\frac{13^2}{5^2} - 1} = 5 \times \frac{12}{5} = 12$$

and centre of hyperbola  $\equiv (5, 0)$

$$\therefore \text{ Required equation of hyperbola is } \frac{(x-5)^2}{25} - \frac{(y-0)^2}{144} = 1$$

**14. (a) :** Foci are (6,4) and (-4,4),  $e = 2$

$$\therefore \text{ Centre is } \left( \frac{6-4}{2}, \frac{4+4}{2} \right) = (1, 4)$$

$$\Rightarrow 6 = 1 + ae \Rightarrow ae = 5 \Rightarrow a = \frac{5}{2} \text{ and } b = \frac{5}{2}(\sqrt{3})$$

$$\text{Hence, the required equation is } \frac{(x-1)^2}{(25/4)} - \frac{(y-4)^2}{(75/4)} = 1$$

$$\text{or } 12x^2 - 4y^2 - 24x + 32y - 127 = 0$$

**15. (a) :** Given equation of hyperbola is

$$9x^2 - 16y^2 + 72x - 32y - 16 = 0$$

$$\Rightarrow 9(x^2 + 8x) - 16(y^2 + 2y) - 16 = 0$$

$$\Rightarrow 9(x+4)^2 - 16(y+1)^2 = 144$$

$$\Rightarrow \frac{(x+4)^2}{16} - \frac{(y+1)^2}{9} = 1$$

$$\text{Therefore, latus rectum} = \frac{2b^2}{a} = 2 \times \frac{9}{4} = \frac{9}{2}$$

**16. (a) :** The line through (6,2) is

$$y - 2 = m(x - 6) \Rightarrow y = mx + 2 - 6m$$

Now from condition of tangency,

$$(2 - 6m)^2 = 25m^2 - 16$$

$$\Rightarrow 36m^2 + 4 - 24m - 25m^2 + 16 = 0$$

$$\Rightarrow 11m^2 - 24m + 20 = 0$$

If  $m_1, m_2$  are its roots then

$$m_1 + m_2 = \frac{24}{11} \text{ and } m_1 m_2 = \frac{20}{11}$$

**17. (d) :** The given ellipse is  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ . The value

of the expression  $\frac{x^2}{9} + \frac{y^2}{4} - 1$  is positive for  $x = 1$ ,

$y = 2$  and negative for  $x = 2, y = 1$ . Therefore  $P$  lies outside  $E$  and  $Q$  lies inside  $E$ . The value of the expression  $x^2 + y^2 - 9$  is negative for both the points  $P$  and  $Q$ . Therefore  $P$  and  $Q$  both lie inside  $C$ . Hence  $P$  lies inside  $C$  but outside  $E$ .

**18. (b) :** Any normal to the hyperbola is

$$\frac{ax}{\sec \theta} + \frac{by}{\tan \theta} = a^2 + b^2 \quad \dots(i)$$

$$\text{But it is given by } lx + m'y - n = 0 \quad \dots(ii)$$

Comparing (i) and (ii), we get

$$\sec \theta = \frac{a}{l} \left( \frac{n}{a^2 + b^2} \right) \text{ and } \tan \theta = \frac{b}{m'} \left( \frac{n}{a^2 + b^2} \right)$$

$$\text{Hence eliminating } \theta, \text{ we get } \frac{a^2}{l^2} - \frac{b^2}{m'^2} = \frac{(a^2 + b^2)^2}{n^2} \quad \dots(iii)$$

$$\text{Since } a^2 = 16, b^2 = 9, l = m, m' = -1 \text{ and } n = \frac{25\sqrt{3}}{3}$$

$$\therefore \text{ On substituting in (iii), we get } m = \pm \frac{2}{\sqrt{3}}$$

**19. (c) :** Solving equations  $x^2 + y^2 = 5$  and  $y^2 = 4x$

we get,  $x^2 + 4x - 5 = 0$  i.e.,  $x = 1, -5$

$$\text{For } x = 1; y^2 = 4 \Rightarrow y = \pm 2$$

$$\text{For } x = -5; y^2 = -20 \text{ (imaginary values)}$$

$$\therefore \text{ Points are } (1, 2)(1, -2)$$

$$m_1 \text{ for } x^2 + y^2 = 5 \text{ at } (1, 2) \text{ is}$$

$$\frac{dy}{dx} = -\frac{x}{y} \Big|_{(1,2)} = -\frac{1}{2}$$

$$\text{Similarly, } m_2 \text{ for } y^2 = 4x \text{ at } (1, 2) \text{ is } 1$$

$$\therefore \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{-\frac{1}{2} - 1}{1 - \frac{1}{2}} \right| = 3$$

$$\text{20. (c) : } \therefore \frac{2b^2}{a^2} = 9 \Rightarrow 2b^2 = 9a^2 \quad \dots(i)$$

$$\text{Now } b^2 = a^2(e^2 - 1) = \frac{9}{16}a^2 \Rightarrow a = \frac{4}{3}b \quad \dots(ii)$$

$$\left( \therefore e = \frac{5}{4} \right)$$

From (i) and (ii),  $b = 6, a = 8$

$$\text{Hence, equation of hyperbola } \frac{x^2}{64} - \frac{y^2}{36} = 1$$

**21. (a, b) :**  $\frac{x^2}{8} + \frac{y^2}{2} = 1, y^2 = 4x$

Any tangent to parabola is  $y = mx + \frac{1}{m}$

If this line is tangent to ellipse then

$$\frac{1}{m^2} = 8m^2 + 2 \Rightarrow 8m^4 + 2m^2 - 1 = 0$$

$$\therefore m^2 = \frac{-2 \pm \sqrt{4+32}}{16} = \frac{-2 \pm 6}{16}$$

$$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm \frac{1}{2}$$

$$y = \frac{x}{2} + 2 \text{ or } y = -\frac{x}{2} - 2$$

$$x - 2y + 4 = 0 \text{ or } x + 2y + 4 = 0$$

**22. (b, c) :**  $y^2 = 32x$

Let equation of tangent be  $y = mx + \frac{8}{m}$

$$\frac{64}{m^2} = \frac{8}{9}m^2 - \frac{8}{9}$$

$$m = \pm 3, y = \pm 3x \pm 8/3$$

**23. (b, c, d) :** Ellipse is  $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$$\text{Here, } \frac{3}{4} = 1 - e^2 \Rightarrow e = \frac{1}{2}$$

Foci are  $(\pm 1, 0)$

Now the hyperbola is having same focus i.e.  $(\pm 1, 0)$

Let  $e_1$  be the eccentricity of hyperbola

$$2ae_1 = 2$$

$$\text{But } 2a = \frac{1}{2} \text{ So, } e_1 = 4$$

$$b^2 = a^2(e^2 - 1) = \frac{1}{16}(16 - 1) = \frac{15}{16}$$

So, the equation of the hyperbola is

$$\frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{15}{16}} = 1 \Rightarrow \frac{x^2}{1} - \frac{y^2}{15} = \frac{1}{16}$$

Its distance between the directrices

$$= \frac{2a}{e_1} = \frac{1}{2 \times 4} = \frac{1}{8} \text{ units}$$

$$\therefore \text{Length of latusrectum} = \frac{2b^2}{a} = \frac{2 \times 15 \times 4}{16 \times 1} = \frac{15}{2} \text{ units}$$

**(24 - 26) :**

**24. (c) 25. (d) 26. (a)**

$$x^2 + y^2 = a^2; x^2 + y^2 = b^2; b > a > 0, A = (-a, 0); B = (a, 0)$$

Let  $(h, k)$  be a point on the locus.

Any tangent to circle  $x^2 + y^2 = b^2$  is  $x \cos \theta + y \sin \theta = b$

$\therefore$  Equation of parabola is

$$\sqrt{(x-h)^2 + (y-k)^2} = |x \cos \theta + y \sin \theta - b|$$

$$\text{i.e., } (x-h)^2 + (y-k)^2 = (x \cos \theta + y \sin \theta - b)^2$$

The points  $(\pm a, 0)$  satisfy this equation

$$\therefore (a-h)^2 + k^2 = (a \cos \theta - b)^2 \quad \dots(1)$$

$$(a+h)^2 + k^2 = (a \cos \theta + b)^2 \quad \dots(2)$$

Subtracting (1) from (2), we get  $h = b \cos \theta$

$$\therefore \text{Required locus is } (a+x)^2 + y^2 = \left(\frac{ax}{b} + b\right)^2$$

$$\text{i.e., } \frac{x^2}{b^2} + \frac{y^2}{b^2 - a^2} = 1 \text{ which is an ellipse.}$$

**27. (c) :**  $\frac{PS}{\sin \beta} = \frac{PS'}{\sin \alpha} = \frac{2ae}{\sin(\pi - (\alpha + \beta))}$

$$\text{or } \frac{2a}{\sin \alpha + \sin \beta} = \frac{2ae}{\sin(\alpha + \beta)}$$

$$\Rightarrow \frac{1}{e} = \frac{2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}}{2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha + \beta}{2}}$$

$$\therefore \frac{1-e}{1+e} = \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$$

**28 (a) :**  $y - 0 = \tan \frac{\beta}{2}(x + ae) \quad \dots (i)$

$$y - 0 = -\tan \frac{\alpha}{2}(x - ae) \quad \dots (ii)$$

$$\text{or, } y^2 = -\left(\frac{1-e}{1+e}\right)[x^2 - a^2 e^2]$$

$$\Rightarrow \left(\frac{1-e}{1+e}\right)x^2 + y^2 = \left(\frac{1-e}{1+e}\right)a^2 e^2 \text{ or, } \frac{x^2}{a^2 e^2} + \frac{y^2}{\left(\frac{1-e}{1+e}\right)a^2 e^2} = 1$$

which is clearly an ellipse.

**29. (b) :**  $e' = \sqrt{1 - \frac{1-e}{1+e}} = \sqrt{\frac{2e}{1+e}}$

**30. (b) :** Any point on the hyperbola  $xy = 16$  is  $\left(4t, \frac{4}{t}\right)$  of the normal passes through  $P(h, k)$ , then

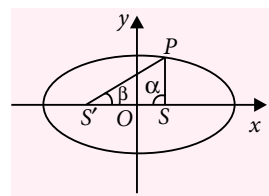
$$k - 4/t = t^2(h - 4t)$$

$$\Rightarrow 4t^4 - t^3 h + tk - 4 = 0$$

$$\therefore \sum t_1 = \frac{h}{4} \text{ and } \sum t_1 t_2 = 0$$

$$\sum t_1 t_2 t_3 = -\frac{k}{4} \text{ and } t_1 t_2 t_3 t_4 = -1$$

$$\therefore \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} = \frac{k}{4} \Rightarrow y_1 + y_2 + y_3 + y_4 = k$$



Now,  $t_1^2 + t_2^2 + t_3^2 + t_4^2 = \frac{h^2}{16} = k$

$\Rightarrow$  Locus of  $(h, k)$  is  $x^2 = 16y$ .

**31. (c) :**  $x^2 = 16y$

Equation of tangent of  $P$  is

$$x \cdot 4t = \frac{16(y+t^2)}{2}$$

$$4tx = 8y + 8t^2$$

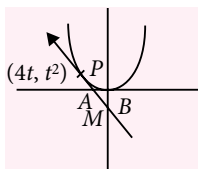
$$tx = 2y + 2t^2$$

$$A = (2t, 0), B = (0, -t^2)$$

$M(h, k)$  is the middle point of  $AB$

$$h = t, k = -\frac{t^2}{2} \Rightarrow 2k = -h^2$$

Locus of  $M(h, k)$  is  $x^2 + 2y = 0$ .

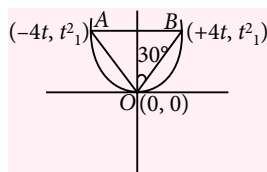


**32. (d) :**  $\tan 30^\circ = \frac{4t_1}{t_1^2} = \frac{4}{t_1}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{t_1} \Rightarrow t_1 = 4\sqrt{3}$$

$$AB = 8t_1 = 32\sqrt{3}$$

$$\text{Area of } \Delta OAB = \frac{\sqrt{3}}{4} \times 32\sqrt{3} \times 32\sqrt{3} = 768\sqrt{3} \text{ sq. units}$$



**33. A-Q, B-Q, C-P**

(A) Equation of tangent at  $(3, 6) : y = x + 3$

$\therefore T(-3, 0)$

Equation of normal at  $(3, 6) : y = -x + 9 \therefore G(9, 0)$

Hence middle point is  $(3, 0)$

(B) Point is obviously focus  $(3, 0)$

(C) Let  $A(t_1)$  and  $B(t_2)$ ,  $C(t_3)$  and  $D(t_4)$

If  $AB$  and  $CD$  intersect at a point  $E$  on the axis, then by solving the equations of  $AB$  and  $CD$  we get the relation  $t_1 t_2 = t_3 t_4$

Now equations of the circles with  $AB$  and  $CD$  as diameters are

$$(x - at_1^2)(x - at_2^2) + (y - 2at_1)(y - 2at_2) = 0$$

$$(x - at_3^2)(x - at_4^2) + (y - 2at_3)(y - 2at_4) = 0$$

If we solve these two circles, then the equation of their radical axis is of the form  $y = mx$ . So it passes through the origin.

**34. (1) :** Equation of the normal is  $y = mx - 2m - m^3$

If it pass through  $(21, 30)$  we have  $30 = 21m - 2m - m^3$   
 $\Rightarrow m^3 - 19m + 30 = 0$

Then  $m = -5, 2, 3$

But if  $m = 2$  or  $3$  then the point where the normal

meets the curve will be  $(am^2, -2am)$  where the curve does not exist. Therefore  $m = -5$

$$\therefore m + 6 = 1$$

**35. (1) :** Since  $F_1$  and  $F_2$  are the ends of the diameter

$$\text{Area of } \Delta PF_1F_2 = \frac{1}{2}(F_1P)(F_2P) = \frac{1}{2}x(17-x) = 30$$

$$\Rightarrow x = 5 \text{ or } 12 \Rightarrow F_1F_2 = 13 \therefore K = 1$$

**36. (7) :** Let  $P(4, 0)$  and  $Q(0, 3)$  are two points on given ellipse  $E_1$

$P_1$  and  $Q_1$  are images of  $P, Q$  w.r.t  $x - y - 2 = 0$

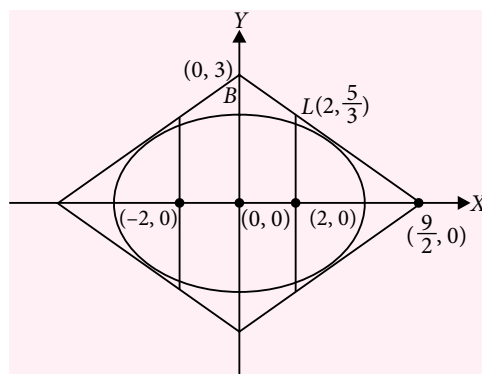
$$\therefore P_1(2, 2) \quad Q_1(5, -2) \text{ lies on } E_2$$

$$\therefore a = -160, b = 292$$

$$\Rightarrow a + b - 125 = 7$$

**37. (9) :** Eccentricity of given ellipse  $e = 2/3$

Equation of tangent at  $L$  is  $\frac{2x}{9} + \frac{y}{3} = 1$ . It meets



$x$ -axis at  $A\left(\frac{9}{2}, 0\right)$  and  $y$  axis at  $B(0, 3)$

$$\therefore \text{Area} = 4 \left[ \frac{1}{2} \cdot \frac{9}{2} \cdot 3 \right] = 27 \therefore K = 9.$$

**38. (8) :**  $xy + 2x + 4y + C = 0$  represents pair of lines  
 $\Rightarrow C = 8$

**39. (5) :** Equation can be rewritten as

$$\sqrt{(x-2)^2 + (y+3)^2} = \frac{13}{5} \left| \frac{12x-5y+1}{13} \right| \text{ So, } e = \frac{13}{5}.$$

$$\therefore \frac{25e}{13} = 5$$

$$\mathbf{40. (2) : 2 \tan^{-1} \left( \frac{b}{a} \right) = \frac{\pi}{3}}$$

$$\frac{b}{a} = \frac{1}{\sqrt{3}}, e^2 = 1 + \frac{1}{3} = \frac{4}{3}$$

Let eccentricity of conjugate hyperbola be  $e'$ .

$$\therefore \frac{1}{e'^2} + \frac{1}{e^2} = 1 \Rightarrow \frac{1}{e'^2} + \frac{3}{4} = 1 \Rightarrow \frac{1}{e'^2} = \frac{1}{4} \Rightarrow e' = 2$$



# MPP-8 MONTHLY Practice Problems

Class XI



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## STATISTICS AND PROBABILITY

Total Marks : 80

Time Taken : 60 Min.

### Only One Option Correct Type

1. A five digit number be selected at random. The probability that the digits in the odd places are odd and the even places are even (repetition is not allowed)

(a)  $\frac{{}^5P_2 \times {}^5P_3}{10^4 \times 9}$  (b)  $\frac{{}^5P_2 \times {}^5P_3}{10^5}$   
 (c)  $\frac{{}^5C_2 \times {}^5C_3}{10^4 \times 9} \times 2$  (d)  $\frac{{}^5C_2 \times {}^5C_3}{9 \times 10^4}$

2. The mean of the data is 30 and data are given as

C.I.	0-10	10-20	20-30	30-40	40-50
Frequency	8	10	$f_1$	15	$f_2$

If total of frequencies is 70, then the missing numbers are

- (a) 14, 23 (b) 25, 21  
 (c) 24, 20 (d) none of these
3. The probability that A speaks truth is  $\frac{4}{5}$ , B speaks truth is  $\frac{3}{4}$ . The probability they contradict each other is

(a)  $\frac{7}{20}$  (b)  $\frac{1}{5}$  (c)  $\frac{3}{20}$  (d)  $\frac{4}{5}$

4. In a series there are  $2n$  observations half of them equal to 'a' and half equal to  $-a$ . If the standard deviation of the observations is 2, then  $|a|$  equals

(a) 2 (b)  $\sqrt{2}$  (c)  $\frac{1}{n}$  (d)  $\frac{\sqrt{2}}{n}$

5. If ten rupee coins which are 10 in number and five rupee coins which are 5 in number are to be placed in a line, then the probability that the extreme coins are five rupee coins is

(a)  $\frac{1}{15!}$  (b)  $\frac{1}{10!}$   
 (c)  $\frac{5!10!}{15!}$  (d) none of these

6. In an experiment with 15 observations on  $x$ , the following results were available :  $\Sigma x^2 = 2830$  and  $\Sigma x = 170$ . One observation i.e., + 20 was found to be wrong and was replaced by the correct value 30, then the corrected variance is

(a) 188.66 (b) 177.33 (c) 8.33 (d) 78.00

### One or More Than One Option(s) Correct Type

7. The probabilities that a student pass in Mathematics, Physics and Chemistry are  $\alpha$ ,  $\beta$  and  $\gamma$  respectively. Of these subjects, student has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two and a 40% chance of passing in exactly two subjects. Which of the following relations are true?

(a)  $\alpha + \beta + \gamma = 19/20$  (b)  $\alpha + \beta + \gamma = 27/20$   
 (c)  $\alpha\beta\gamma = 1/10$  (d)  $\alpha\beta\gamma = 1/4$

8. The variable  $x$  takes two values  $x_1$  and  $x_2$  with frequencies  $f_1$  and  $f_2$ , respectively. If  $\sigma$  denotes the standard deviation of  $x$ , then

(a)  $\sigma^2 = \frac{f_1x_1^2 + f_2x_2^2}{f_1 + f_2} - \left( \frac{f_1x_1 + f_2x_2}{f_1 + f_2} \right)^2$

(b)  $\sigma^2 = \frac{f_1f_2}{(f_1 + f_2)^2} (x_1 - x_2)^2$

(c)  $\sigma^2 = \frac{(x_1 - x_2)^2}{f_1 + f_2}$  (d) None of these

9. If A and B are two events such that  $P(A) = 1/2$  and  $P(B) = 2/3$ , then

(a)  $P(A \cup B) \geq 2/3$  (b)  $P(A \cap B') \leq 1/3$   
 (c)  $1/6 \leq P(A \cap B) \leq 1/2$  (d)  $1/6 \leq P(A \cap B) \leq 1/2$

10. For two data sets, each of size 5, the variance are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(a)  $\frac{5}{2}$  (b)  $\frac{11}{2}$  (c) 6 (d)  $\frac{13}{2}$

11. The letters of the word PROBABILITY are written down at random in a row. Let  $E_1$  denotes the event that two I's are together and  $E_2$  denotes the event that two B's are together, then

(a)  $P(E_1) = P(E_2) = \frac{2}{11}$  (b)  $P(E_1 \cap E_2) = \frac{2}{55}$

(c)  $P(E_1 \cup E_2) = \frac{18}{55}$  (d) none of these

12. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test

Marks	0	1	2	3	4	5
Frequency	$x-2$	$x$	$x^2$	$(x+1)^2$	$2x$	$x+1$

where,  $x$  is a positive integer. Then,

- (a) Mean = 2.8 (b) Variance = 1.12  
(c) Standard Deviation = 1.12  
(d) None of these
13. For two events  $A$  and  $B$ ,  $P(A \cap B)$  is
- (a) not less than  $P(A) + P(B) - 1$   
(b) not greater than  $P(A) + P(B)$   
(c) equal to  $P(A) + P(B) - P(A \cup B)$   
(d) equal to  $P(A) + P(B) + P(A \cup B)$

### Comprehension Type

A JEE aspirant estimates that she will be successful with an 80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day and with a 40% chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively:

14. Given that she is successful, the probability that she studied for 4 hours, is

(a)  $\frac{6}{12}$  (b)  $\frac{7}{12}$  (c)  $\frac{8}{12}$  (d)  $\frac{9}{12}$

15. Given that she does not achieve success, the probability she studied for 4 hours, is

(a)  $\frac{18}{26}$  (b)  $\frac{19}{26}$  (c)  $\frac{20}{26}$  (d)  $\frac{21}{26}$

### Matrix Match Type

16. Match the following:

Column I		Column II
P.	Two balls are drawn from an urn containing 2 white, 3 red and 4 black balls one by one without replacement. If the probability that both the balls are of same colour is $\lambda$ and at least one ball is red is $\mu$ , then	1. $\mu = \frac{8}{15}$
Q.	A bag contains 2 white and 4 black balls while another bag contains 6 white and 4 black balls. A bag is selected at random and a ball is drawn. If $\lambda$ be the probability that the ball drawn is of white colour and $\mu$ be the probability that the ball drawn is black colour, then	2. $\mu = \frac{11}{12}$
R.	Bag A contains 4 red and 5 black balls and bag B contains 3 red and 7 black balls. One ball is drawn from bag A and two from bag B. If $\lambda$ be the probability that out of 3 balls drawn two are black and one is red and $\mu$ be the probability that out of three balls drawn two are red and one is black, then	3. $\lambda = \frac{5}{18}$
		4. $\mu = \frac{11}{45}$

P	Q	R
(a) 2	1	3
(b) 3	4	2
(c) 3	1	4
(d) 1	2	3

### Integer Answer Type

17. If four squares are chosen at random on a chess board. If the probability that they lie on a diagonal line is  $\frac{xyz}{{}^{64}C_4}$ , then the value of  $x + y - z$  must be
18. If the mean deviation about the median of the numbers  $a, 2a, \dots, 50a$  is 50, then  $|a|$  equals
19. Two integers  $x$  and  $y$  are chosen (without random) at random from the set  $\{x : 0 \leq x \leq 10, x \text{ is an integer}\}$ . If the probability for  $|x - y| \leq 5$  is  $p$ , then the value of  $121p - 91$  must be
20. If the variance of first 50 even natural numbers is  $abc$ . Then find the value of  $a + b - c$ . ♦♦

Keys are published in this issue. Search now! ☺

## SELF CHECK

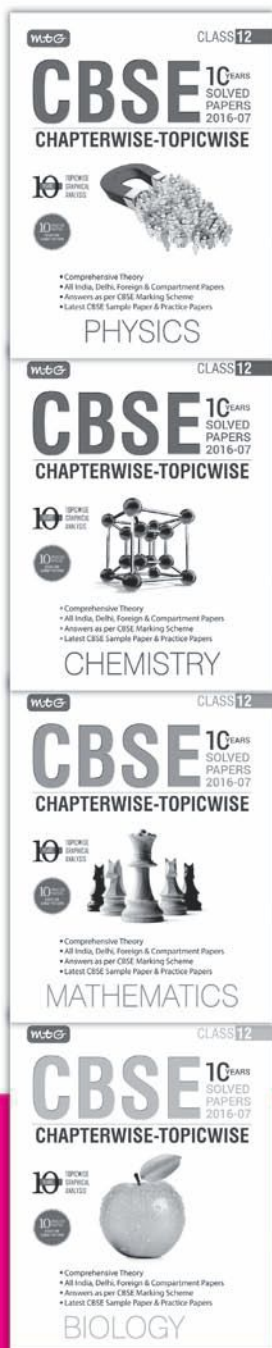
No. of questions attempted .....  
No. of questions correct .....  
Marks scored in percentage .....

### Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.



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- Students don't have to wait till they finish their entire course to use previous years' papers - "one chapter at a time, one topic at a time" is also an option
- Some topics require more focus than the others
- Different type of questions require different type of answers

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# ACE

YOUR WAY **CBSE XII**

## PRACTICE PAPER 2017



Exam on  
20<sup>th</sup> March  
2017

Time Allowed : 3 hours

Maximum Marks : 100

### GENERAL INSTRUCTIONS

- (i) All questions are compulsory.
- (ii) This question paper contains **29** questions.
- (iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.
- (iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.
- (v) Questions **13-23** in Section-C are long-answer-I type questions carrying **4** marks each.
- (vi) Questions **24-29** in Section-D are long-answer-II type questions carrying **6** marks each.

### SECTION-A

1. Write the vector equation of a line passing through the point  $(1, -1, 2)$  and parallel to the line  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ .
2. Evaluate :  $\int \frac{x^3 - x^2 + x - 1}{x-1} dx$
3. If the binary operation  $*$  on the set of integers  $Z$  is defined by  $a * b = a + 3b^2$ , then find the value of  $2 * 4$ .
4. If  $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix} = 8$ , then find the value of  $x$ .

### SECTION-B

5. Prove that :  $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$ .
6. If  $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ , find  $A^2 - 5A + 4I$ .

7. Show that  $f: N \rightarrow N$ , defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

is a many-one onto function.

8. For what value of  $k$ , the function defined by

$$f(x) = \begin{cases} k(x^2 + 2), & \text{if } x \leq 0 \\ 3x + 1, & \text{if } x > 0 \end{cases}$$

is continuous at  $x = 0$ ?

9. Form the differential equation of the family of curves  $y = a \cos(x + b)$ , where  $a$  and  $b$  are arbitrary constants.
10. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.
11. Let  $*$  be a binary operation on the set of rational numbers given as  $a * b = (2a - b)^2$ ,  $a, b \in Q$ . Find  $3 * 5$  and  $5 * 3$ . Is  $3 * 5 = 5 * 3$ ?
12. If  $xy + y^2 = \tan x + y$ , find  $\frac{dy}{dx}$ .

### SECTION-C

13. Using integration, find the area of the region bounded by the curves  $y = x^2$  and  $y = x$ .
14. Solve the differential equation  $\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y^2 = 0$ .
15. In a parliament election, a political party hired a public relations firm to promote its candidates in three ways-telephone, house calls and letters. The cost per contact (in paise) is given in matrix  $A$  as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{matrix} \text{Telephone} \\ \text{House call} \\ \text{Letters} \end{matrix}$$

The number of contacts of each type made in two cities  $X$  and  $Y$  is given in matrix  $B$  as

$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{matrix} \text{City } X \\ \text{City } Y \end{matrix}$$

Find the total amount spent by the party in the two cities. What should one consider before casting his/her vote-party's promotional activity or their social activities?

OR

If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , then verify that  $A^2 - 4A - 5I = O$ .

Hence find  $A^{-1}$ .

16. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are three vectors such that each one is perpendicular to the vector obtained by sum of the other two and  $|\vec{a}| = 3$ ,  $|\vec{b}| = 4$  and  $|\vec{c}| = 5$ , then prove that  $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$ .
17. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.
18. If  $y = 3\cos(\log x) + 4\sin(\log x)$ , then show that

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

OR

For what value of  $\lambda$ , the function defined by  $f(x) = \begin{cases} \lambda(x^2 + 2), & \text{if } x \leq 0 \\ 4x + 6, & \text{if } x > 0 \end{cases}$  is continuous at  $x = 0$ ? Hence check the differentiability of  $f(x)$  at  $x = 0$ .

19. Find the equation of the plane passing through the intersection of the planes  $x + 3y + 6 = 0$  and  $3x - y - 4z = 0$  and whose perpendicular distance from the origin is unity.

20. Evaluate:  $\int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} dx$

OR

Using properties of definite integrals prove that

$$\int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$$

21. Prove that:

$$\frac{9\pi}{8} - \frac{9}{4} \sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4} \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right).$$

22. Find the intervals in which the following function is (a) increasing (b) decreasing:

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

23. Using properties of determinants, show that

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$$

### SECTION-D

24. Find the equation of the plane through the line of intersection of the planes

$$2x + y - z = 3 \text{ and } 5x - 3y + 4z + 9 = 0$$

and parallel to the line  $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$ .

Also, find the distance of the plane from origin.

OR

Find the equation of the plane passing through the point  $A(1, 2, 1)$  and perpendicular to the line joining the points  $P(1, 4, 2)$  and  $Q(2, 3, 5)$ . Also, find the distance of this plane from the line

$$\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-7}{-1}.$$

25. Find the particular solution of the differential equation  $x \frac{dy}{dx} + y - x + xy \cot x = 0$ ;  $x \neq 0$ , given that when  $x = \frac{\pi}{2}$ ,  $y = 0$ .

26. If  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  are two square matrices, find  $AB$  and hence solve the system of equations

$$x - y = 3, 2x + 3y + 4z = 17 \text{ and } y + 2z = 7.$$

OR

Two schools  $P$  and  $Q$  want to award their selected students on the values of discipline, politeness and punctuality. The school  $P$  wants to award ₹  $x$  each, ₹  $y$  each and ₹  $z$  each for the three respective values to its 3, 2 and 1 students with a total award money of ₹ 1,000. School  $Q$  wants to spend ₹ 1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is ₹ 600, using matrices, find the award money for each value.

27. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items. An electronic sewing machine costs him ₹ 360 and a manually operated sewing machine costs ₹ 240. He can sell an electronic sewing machine at a profit of ₹ 22 and a manually operated sewing machine at a profit of ₹ 18. Assuming that, he can sell all the items that he can buy, how should he invest his money in order to maximize his profit. Make it as a linear programming problem and solve it graphically. Keeping the rural background in mind justify the 'values' to be promoted for the selection of manually operated machine.

28. If the length of three sides of a trapezium other than base is 10 cm each, then find the area of the trapezium when it is maximum.

29. From a well shuffled pack of 52 cards, 3 cards are drawn one-by-one with replacement. Find the probability distribution of number of queens.

OR

Suppose the probability for  $A$  to win a game against  $B$  is 0.4. If  $A$  has an option of playing a 'best of 3 games' or a 'best of 5 games' match against  $B$ , which option should  $A$  choose so that the probability of his winning the match is higher? (No game ends in draw).

## SOLUTIONS

1. Vector equation of the line passing through  $(1, -1, 2)$  and parallel to the line  $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$  is  

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$$

2. Let  $I = \int \frac{x^3 - x^2 + x - 1}{x - 1} dx$   

$$= \int \frac{x^2(x-1) + 1(x-1)}{x-1} dx = \int \frac{(x^2+1)(x-1)}{x-1} dx$$
  

$$= \int (x^2+1) dx = \frac{1}{3}x^3 + x + C$$

3. Here,  $a * b = a + 3b^2$   
 $\therefore 2 * 4 = 2 + 3(4)^2 = 2 + 3 \times 16 = 50$

4. Given,  $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix} = 8$   

$$\Rightarrow x(-x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta) = 8$$
  

$$\Rightarrow -x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta - \sin \theta \cos \theta + x \cos^2 \theta = 8$$
  

$$\Rightarrow -x^3 - x + x(\sin^2 \theta + \cos^2 \theta) = 8$$
  

$$\Rightarrow -x^3 - x + x = 8 \Rightarrow x^3 + 8 = 0$$
  

$$\Rightarrow (x+2)(x^2 - 2x + 4) = 0 \Rightarrow x + 2 = 0$$
  

$$[\because x^2 - 2x + 4 > 0 \forall x]$$
  

$$\Rightarrow x = -2$$

5. L.H.S.  $= \left( \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} \right) + \left( \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} \right)$   

$$= \tan^{-1} \left( \frac{\frac{1}{3} + \frac{1}{5}}{1 - \frac{1}{3} \times \frac{1}{5}} \right) + \tan^{-1} \left( \frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7} \times \frac{1}{8}} \right)$$
  

$$= \tan^{-1} \left( \frac{\frac{8}{15}}{\frac{14}{15}} \right) + \tan^{-1} \left( \frac{\frac{15}{56}}{\frac{55}{56}} \right)$$
  

$$= \tan^{-1} \left( \frac{4}{7} \right) + \tan^{-1} \left( \frac{3}{11} \right)$$
  

$$= \tan^{-1} \left[ \frac{\frac{4}{7} + \frac{3}{11}}{1 - \frac{4}{7} \times \frac{3}{11}} \right] = \tan^{-1} \left( \frac{\frac{65}{77}}{\frac{65}{77}} \right)$$
  

$$= \tan^{-1} 1 = \frac{\pi}{4} = \text{R.H.S.}$$

6. We have,  $A^2 - 5A + 4I$   

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

7. We have,  $f(1) = \frac{(1+1)}{2} = \frac{2}{2} = 1$  and  $f(2) = \frac{2}{2} = 1$

Thus,  $f(1) = f(2)$  while  $1 \neq 2$

$\therefore f$  is many-one.

In order to show that  $f$  is onto, consider an arbitrary element  $n \in N$ .

If  $n$  is odd, then  $(2n-1)$  is odd

$$\therefore f(2n-1) = \frac{(2n-1+1)}{2} = \frac{2n}{2} = n$$

If  $n$  is even, then  $2n$  is even

$$\therefore f(2n) = \frac{2n}{2} = n$$

Thus, for each  $n \in N$  (whether even or odd) there exists its pre-image in  $N$ .

$\therefore f$  is onto.

Hence,  $f$  is many-one onto function.

8. We have,  $\lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} k(h^2 + 2) = 2k$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} (3h + 1) = 1 \text{ and } f(0) = 2k$$

As  $f(x)$  is continuous at  $x = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\Rightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$$

9. Here,  $y = a \cos(x + b)$  ... (i)

Differentiating (i) w.r.t.  $x$ , we get

$$\frac{dy}{dx} = -a \sin(x + b)$$

Again differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = -a \cos(x + b)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y \Rightarrow \frac{d^2y}{dx^2} + y = 0.$$

10. Let  $r$  be the radius of sphere and  $\Delta r$  be the error in measuring the radius. Then,  $r = 9$  cm,  $\Delta r = 0.03$  cm  
Now, surface area ( $S$ ) of the sphere is given by  $4\pi r^2$

$$\Rightarrow \frac{dS}{dr} = 8\pi r$$

$$\therefore \Delta S = \frac{dS}{dr} \Delta r = 8\pi r \Delta r$$

$$= 8\pi \times 9 \times 0.03 = 2.16 \pi \text{ cm}^2$$

This is the approximate error in calculating surface area.

11. We have,  $a * b = (2a - b)^2$

$$\therefore 3 * 5 = (2 \times 3 - 5)^2 = (6 - 5)^2 = 1$$

$$5 * 3 = (2 \times 5 - 3)^2 = (10 - 3)^2 = 49$$

Thus,  $3 * 5 \neq 5 * 3$

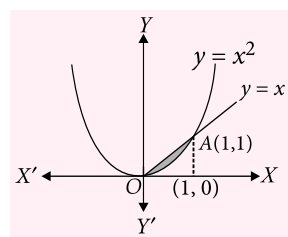
12. Given,  $xy + y^2 = \tan x + y$

Differentiating w.r.t.  $x$ , we get

$$x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} (x + 2y - 1) = \sec^2 x - y \Rightarrow \frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$$

13. The given curves are  $y = x^2$  ... (i) and  $y = x$  ... (ii)



The point of intersection of (i) and (ii) are  $A(1, 1)$  and  $O(0, 0)$ .

$\therefore$  Required area = Area of shaded region

$$= \int_0^1 (x - x^2) dx = \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \left[ \frac{1}{2} - \frac{1}{3} \right] = \frac{1}{6} \text{ sq. unit.}$$

14. We have,  $\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y^2 = 0$

$$\frac{\log y}{y^2} dy + \frac{x^2}{\operatorname{cosec} x} dx = 0$$

Integrating both sides, we get

$$\int \frac{\log y}{y^2} dy + \int x^2 \sin x dx = 0$$

$$\text{Put } \log y = t \Rightarrow \frac{1}{y} dy = dt \text{ and } y = e^t$$

### Solution Sender of Maths Musing

#### SET-169

- |                    |            |
|--------------------|------------|
| 1. N. Jayanthi     | Hyderabad  |
| 2. Ravinder Gajula | Karimnagar |

$$\begin{aligned}
&\Rightarrow \int t \cdot e^{-t} dt + \int x^2 \sin x dx = 0 \\
&\Rightarrow t \cdot \frac{e^{-t}}{-1} - \int 1 \cdot \frac{e^{-t}}{-1} dt + x^2(-\cos x) - \int 2x(-\cos x) dx = C \\
&\Rightarrow -t e^{-t} - e^{-t} - x^2 \cos x + 2x \sin x - 2 \int 1 \cdot \sin x dx = C \\
&\Rightarrow -\frac{1 + \log y}{y} - x^2 \cos x + 2x \sin x + 2 \cos x = C
\end{aligned}$$

is the required solution.

15. The total amount spent by the party in two cities X and Y is represented in the matrix equation by matrix C as,

$$\begin{aligned}
C &= BA \\
\Rightarrow \begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} 1000 \times 140 + 500 \times 200 + 5000 \times 150 \\ 3000 \times 140 + 1000 \times 200 + 10000 \times 150 \end{bmatrix} \\
&= \begin{bmatrix} 990000 \\ 2120000 \end{bmatrix}
\end{aligned}$$

$\therefore X = ₹ 990000$  and  $Y = ₹ 2120000$

Thus, amount spent by the party in city X and Y is ₹ 990000 and ₹ 2120000 respectively. One should consider about the social activity before casting his/her vote.

OR

$$\text{Given, } A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\text{Now, } A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$\begin{aligned}
\therefore A^2 - 4A - 5I &= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O
\end{aligned}$$

$$\text{Now, } A^2 - 4A - 5I = O$$

Pre-multiplying by  $A^{-1}$  both sides, we get

$$(A^{-1}A)A - 4(A^{-1}A) - 5(A^{-1}I) = O$$

$$\Rightarrow IA - 4I = 5A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{5}(A - 4I) = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

16. Given,  $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$  ... (i)  
and  $\vec{a} \cdot (\vec{b} + \vec{c}) = 0, \vec{b} \cdot (\vec{c} + \vec{a}) = 0, \vec{c} \cdot (\vec{a} + \vec{b}) = 0$   
 $\therefore \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} = 0$   
 $\Rightarrow 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$  ... (ii)  
Now,  $|\vec{a} + \vec{b} + \vec{c}|^2$   
 $= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b}) + 2(\vec{b} \cdot \vec{c}) + 2(\vec{c} \cdot \vec{a})$   
 $= 3^2 + 4^2 + 5^2 + 0 = 50$  [Using (i) and (ii)]  
 $\therefore |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$ .

17. Here,  $p = P(\text{success of the experiment}) = \frac{2}{2+1} = \frac{2}{3}$   
 $\therefore q = 1 - p = 1 - \frac{2}{3} = \frac{1}{3}$

Also  $n = 6$

Let X denote the number of successes.

$$\begin{aligned}
\therefore \text{Required probability } P(X \geq 4) &= P(X = 4) + P(X = 5) + P(X = 6) \\
&= {}^6C_4 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 + {}^6C_5 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 + {}^6C_6 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 \\
&= 15 \times \frac{16}{3^6} + 6 \times \frac{32}{3^6} + \frac{64}{3^6} = \frac{240 + 192 + 64}{3^6} = \frac{496}{729}
\end{aligned}$$

18. We have,  $y = 3 \cos(\log x) + 4 \sin(\log x)$

Differentiating w.r.t. x, we get

$$\begin{aligned}
\frac{dy}{dx} &= -3 \sin(\log x) \times \frac{1}{x} + 4 \cos(\log x) \times \frac{1}{x} \\
\Rightarrow x \frac{dy}{dx} &= -3 \sin(\log x) + 4 \cos(\log x)
\end{aligned}$$

Again differentiating w.r.t. x, we get

$$\begin{aligned}
x \frac{d^2y}{dx^2} + \frac{dy}{dx} &= -3 \cos(\log x) \times \frac{1}{x} - 4 \sin(\log x) \times \frac{1}{x} \\
\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} &= -[3 \cos(\log x) + 4 \sin(\log x)] \\
\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} &= -y \Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0
\end{aligned}$$

OR

$$\text{Here, } f(x) = \begin{cases} \lambda(x^2 + 2), & \text{if } x \leq 0 \\ 4x + 6, & \text{if } x > 0 \end{cases}$$

$$\text{At } x = 0, f(0) = \lambda(0^2 + 2) = 2\lambda$$

$$\text{L.H.L.} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} \lambda[(-h)^2 + 2] = 2\lambda$$

$$\text{R.H.L.} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} [4(h) + 6] = 6$$

$$\therefore \text{ For } f \text{ to be continuous at } x = 0, 2\lambda = 6 \Rightarrow \lambda = 3.$$

Hence, the function becomes

$$f(x) = \begin{cases} 3(x^2 + 2), & \text{if } x \leq 0 \\ 4x + 6, & \text{if } x > 0 \end{cases}$$

$$f'(0^-) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{0-h}$$

$$= \lim_{h \rightarrow 0} \frac{3(h^2 + 2) - 6}{-h} = \lim_{h \rightarrow 0} (-3h) = 0$$

$$\text{and } f'(0^+) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{0+h} = \lim_{h \rightarrow 0} \frac{4h + 6 - 6}{h} = 4$$

$$\Rightarrow f'(0^-) \neq f'(0^+)$$

$\therefore f$  is not differentiable at  $x = 0$ .

19. Equation of plane passing through intersection of given planes is

$$x + 3y + 6 + \lambda(3x - y - 4z) = 0$$

$$\Rightarrow \vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} - 4\lambda\hat{k}] + 6 = 0$$

$\therefore$  Distance of plane from origin is unity.

$$\therefore \left| \frac{6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + 16\lambda^2}} \right| = 1$$

$$\Rightarrow 26 = 26\lambda^2 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

$\therefore$  Equation of the required plane is

$$\vec{r} \cdot (4\hat{i} + 2\hat{j} - 4\hat{k}) + 6 = 0$$

$$\text{or } \vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) + 6 = 0$$

20. Let  $I = \int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} dx$

$$\text{Put } \sin x = t \Rightarrow \cos x dx = dt$$

$$\therefore I = \int \frac{dt}{(1-t)(2-t)}$$

$$\text{We write, } \frac{1}{(1-t)(2-t)} = \frac{A}{1-t} + \frac{B}{2-t}$$

$$\Rightarrow 1 = A(2-t) + B(1-t) \quad \dots(1)$$

Putting  $t = 1$  in (1), we get

$$1 = A(2-1) \Rightarrow A = 1$$

Putting  $t = 2$  in (1), we get

$$1 = B(1-2) \Rightarrow B = -1$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{1-t} + \int \frac{-dt}{2-t} = -\log|1-t| + \log|2-t| + C \\ &= \log \left| \frac{2-t}{1-t} \right| + C = \log \left| \frac{2 - \sin x}{1 - \sin x} \right| + C \end{aligned}$$

OR

$$\text{Let } I = \int_0^\pi \frac{x \tan x}{\sec x \operatorname{cosec} x} dx \quad \dots(1)$$

$$\text{Using } \int_0^a f(x) dx = \int_0^a f(a-x) dx, \text{ we get}$$

$$I = \int_0^\pi \frac{(\pi-x) \tan(\pi-x)}{\sec(\pi-x) \operatorname{cosec}(\pi-x)} dx = \int_0^\pi \frac{(\pi-x) \tan x}{\sec x \operatorname{cosec} x} dx \quad \dots(2)$$

Adding (1) and (2), we get

$$\begin{aligned} 2I &= \pi \int_0^\pi \frac{\tan x}{\sec x \operatorname{cosec} x} dx = \pi \int_0^\pi \sin^2 x dx \\ &= \pi \int_0^\pi \frac{1 - \cos 2x}{2} dx = \frac{\pi}{2} \left[ x - \frac{\sin 2x}{2} \right]_0^\pi = \frac{\pi^2}{2} \end{aligned}$$

$$\Rightarrow I = \frac{\pi^2}{4}$$

$$\begin{aligned} 21. \text{ L.H.S.} &= \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left( \frac{1}{3} \right) = \frac{9}{4} \left[ \frac{\pi}{2} - \sin^{-1} \left( \frac{1}{3} \right) \right] \\ &= \frac{9}{4} \cos^{-1} \left( \frac{1}{3} \right) \quad \left[ \because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right] \\ &= \frac{9}{4} \sin^{-1} \sqrt{1 - \frac{1}{9}} \quad \left[ \because \cos^{-1} x = \sin^{-1} \sqrt{1 - x^2} \right] \\ &\quad \text{for } 0 \leq x \leq 1 \\ &= \frac{9}{4} \sin^{-1} \sqrt{\frac{8}{9}} = \frac{9}{4} \sin^{-1} \left( \frac{2\sqrt{2}}{3} \right) = \text{R.H.S.} \end{aligned}$$

22. The given function is

$$\begin{aligned} f(x) &= x^4 - 8x^3 + 22x^2 - 24x + 21 \\ \Rightarrow f'(x) &= 4x^3 - 24x^2 + 44x - 24 \\ &= 4(x^3 - 6x^2 + 11x - 6) \\ &= 4(x-1)(x^2 - 5x + 6) \\ &= 4(x-1)(x-2)(x-3) \end{aligned}$$

Thus  $f'(x) = 0 \Rightarrow x = 1, 2, 3$ .

Hence, possible disjoint intervals are

$(-\infty, 1)$ ,  $(1, 2)$ ,  $(2, 3)$  and  $(3, \infty)$ .

In the interval  $(-\infty, 1)$ ,  $f'(x) < 0$

In the interval  $(1, 2)$ ,  $f'(x) > 0$

In the interval  $(2, 3)$ ,  $f'(x) < 0$

In the interval  $(3, \infty)$ ,  $f'(x) > 0$

$\therefore f$  is increasing in  $(1, 2) \cup (3, \infty)$  and  $f$  is decreasing in  $(-\infty, 1) \cup (2, 3)$ .

23. L.H.S. =  $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$ .

Since each element in the first column of determinant is the sum of two elements, therefore, determinant can be expressed as the sum of two determinants given by

$$\begin{vmatrix} x & x & x \\ 5x & 4x & 2x \\ 10x & 8x & 3x \end{vmatrix} + \begin{vmatrix} y & x & x \\ 4y & 4x & 2x \\ 8y & 8x & 3x \end{vmatrix}$$

Taking  $x$  common from  $R_1, R_2, R_3$  in first determinant and  $x$  common from  $C_2, C_3, y$  common from  $C_1$  in second determinant, we get

$$x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^2 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 2 \\ 8 & 8 & 3 \end{vmatrix} = x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^2 \cdot 0$$

( $\because C_1$  and  $C_2$  are identical in the second determinant)

Applying  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$ , we get

$$x^3 \begin{vmatrix} 0 & 0 & 1 \\ 3 & 2 & 2 \\ 7 & 5 & 3 \end{vmatrix} = x^3 \cdot 1 \cdot (15 - 14) = x^3 = \text{R.H.S.}$$

24. The equation of the plane passing through the line of intersection of the planes  $2x + y - z = 3$  and  $5x - 3y + 4z + 9 = 0$  is

$$(2x + y - z - 3) + \lambda(5x - 3y + 4z + 9) = 0$$

$$\Rightarrow x(2 + 5\lambda) + y(1 - 3\lambda) + z(4\lambda - 1) + 9\lambda - 3 = 0 \quad \dots(i)$$

Since, plane (i) is parallel to the line

$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$$

$$\therefore 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$$

$$\Rightarrow 18\lambda + 3 = 0 \Rightarrow \lambda = -\frac{1}{6}$$

Putting the value of  $\lambda$  in (i), we obtain

$$7x + 9y - 10z - 27 = 0$$

This is the equation of the required plane.

Now, distance of the plane  $7x + 9y - 10z - 27 = 0$  from the origin is,

$$\left| \frac{-27}{\sqrt{7^2 + 9^2 + 10^2}} \right| = \frac{27}{230} \text{ unit}$$

OR

The line joining the given points

$P(1, 4, 2)$  and  $Q(2, 3, 5)$  has direction ratios  $\langle 1, -2, 4 \rangle$  i.e.,  $\langle -1, 1, -3 \rangle$

The plane through  $(1, 2, 1)$  and perpendicular to the line  $PQ$  is

$$-1(x - 1) + 1(y - 2) - 3(z - 1) = 0$$

$$\Rightarrow x - y + 3z - 2 = 0$$

Now, direction ratios of line  $\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-7}{-1}$  are  $2, -1, -1$ .

Now,  $2(1) + (-1)(-1) + (3)(-1) = 2 + 1 - 3 = 0$

$\therefore$  Line is parallel to the plane.

Since,  $(-3, 5, 7)$  lies on the given line.

$\therefore$  Distance of the point  $(-3, 5, 7)$  from plane is

$$d = \left| \frac{-3 - 5 + 3(7) - 2}{\sqrt{1 + 1 + 9}} \right| \Rightarrow d = \frac{11}{\sqrt{11}} = \sqrt{11} \text{ units.}$$

25. We have,  $x \frac{dy}{dx} + y - x + xy \cot x = 0, (x \neq 0)$

$$\Rightarrow x \frac{dy}{dx} + (1 + x \cot x) \cdot y = x$$

$$\Rightarrow \frac{dy}{dx} + \frac{1 + x \cot x}{x} \cdot y = 1 \quad \dots(i)$$

This is linear D.E. of the form  $\frac{dy}{dx} + Py = Q$

$$\text{where } P = \frac{1 + x \cot x}{x} = \frac{1}{x} + \cot x, Q = 1$$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\log(x \sin x)} = x \sin x$$

$\therefore$  The solution of (i) is

$$y \cdot x \sin x = \int 1 \cdot x \sin x dx + C$$

$$= x(-\cos x) + \int 1 \cdot \cos x dx + C$$

$$\Rightarrow xy \sin x = -x \cos x + \sin x + C \quad \dots(ii)$$

Putting  $x = \frac{\pi}{2}, y = 0$  in (ii), we get

$$0 = -\frac{\pi}{2} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} + C \Rightarrow C = -1$$

$\therefore xy \sin x = \sin x - x \cos x - 1$  is the required particular solution.

26. Here,  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}; B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$$\Rightarrow A \cdot \left( \frac{1}{6} B \right) = I \Rightarrow A \text{ is invertible and } A^{-1} = \frac{1}{6} B$$

Now, the given system of equations is

$$x - y = 3$$

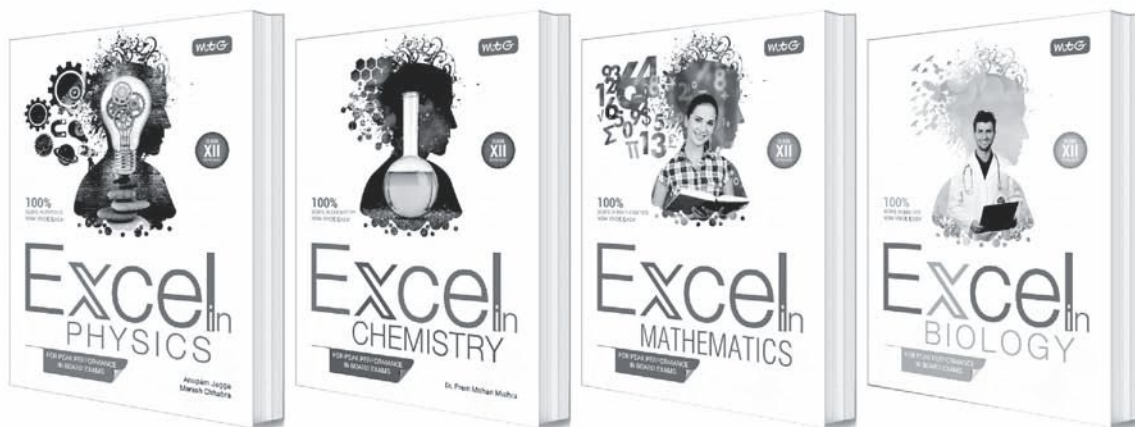
$$2x + 3y + 4z = 17$$

$$y + 2z = 7$$

The system of equations can be written as

$$AX = P$$

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where,  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ ;  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ;  $P = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

Since  $A^{-1}$  exists, so system of equations has a unique solution given by  $X = A^{-1}P$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}BP = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\Rightarrow x = 2, y = -1, z = 4.$$

OR

According to question, we have,

$$3x + 2y + z = 1000$$

...(i)

$$4x + y + 3z = 1500 \quad \dots(ii)$$

$$x + y + z = 600 \quad \dots(iii)$$

The given system of equations can be written as  $AX = B$

where,  $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  and  $B = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -5 \neq 0$$

$\therefore A$  is invertible and system of equations has a unique solution given by  $X = A^{-1}B$

$$\text{Now, } A_{11} = -2, A_{12} = -1, A_{13} = 3,$$

$$A_{21} = -1, A_{22} = 2, A_{23} = -1,$$

$$A_{31} = 5, A_{32} = -5, A_{33} = -5$$

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$$\therefore \text{adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{-1}{5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Now,  $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix} = \frac{-1}{5} \begin{bmatrix} -500 \\ -1000 \\ -1500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

$$\Rightarrow x = 100, y = 200, z = 300$$

Hence, the money awarded for discipline, politeness and punctuality are ₹ 100, ₹ 200 and ₹ 300 respectively.

27. Suppose, number of electronic sewing machines purchased =  $x$  and number of manually operated sewing machines purchased =  $y$

Mathematical formulation of given problem is :

$$\text{Maximize } Z = 22x + 18y$$

Subject to constraints :

$$x + y \leq 20 \quad \dots(i)$$

$$360x + 240y \leq 5760$$

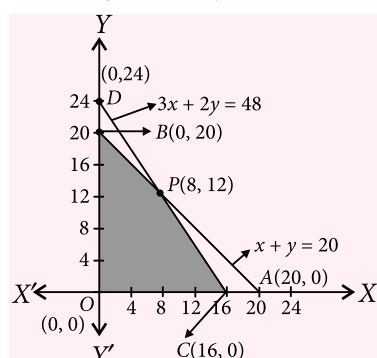
$$\text{or } 3x + 2y \leq 48 \quad \dots(ii)$$

$$x \geq 0, y \geq 0$$

Solving equations  $x + y = 20$  and  $3x + 2y = 48$ , we get  $x = 8, y = 12$

Let  $P \equiv (8, 12)$

Now, we plot the graph of system of inequalities.



The shaded portion  $OCBP$  represents the feasible region which is bounded.

The corner points of feasible region are :  $O(0, 0)$ ,  $C(16, 0)$ ,  $P(8, 12)$  and  $B(0, 20)$ .

Now, we calculate  $Z$  at each corner point.

Corner points	Value of $Z = 22x + 18y$
$O(0, 0)$	$22(0) + 18(0) = 0$
$C(16, 0)$	$22(16) + 18(0) = 352$
$P(8, 12)$	$22(8) + 18(12) = 392$ (Maximum)
$B(0, 20)$	$22(0) + 18(20) = 360$

$\therefore$  Maximum value of  $Z$  is 392 which occurs at the point  $P(8, 12)$ .

Hence, the maximum profit is ₹ 392 when 8 electronic machines and 12 manually operated sewing machines are purchased.

Manually operated machine should be promoted, to save energy and increase employment for rural people.

28. Let  $ABCD$  be the given trapezium.

Then  $AD = DC = CB = 10$  cm

In  $\triangle APD$  and  $\triangle BQC$ ,

$$DP = CQ = h$$

$$AD = BC = 10 \text{ cm}$$

$$\angle DPA = \angle CQB = 90^\circ$$

$$\therefore \triangle APD \cong \triangle BQC \quad (\text{by R.H.S. congruency})$$

$$\Rightarrow AP = QB = x \text{ cm (say)}$$

$$\therefore AB = AP + PQ + QB = x + 10 + x = (2x + 10) \text{ cm}$$

Also from  $\triangle APD$ ,

$$AP^2 + PD^2 = AD^2 \Rightarrow x^2 + h^2 = 10^2$$

$$\Rightarrow h = \sqrt{100 - x^2} \quad \dots(i)$$

Now, area  $S$  of this trapezium is given by

$$S = \frac{1}{2}(AB + DC) \cdot h = \frac{1}{2}(2x + 10 + 10)h = (x + 10) \cdot \sqrt{100 - x^2} \quad \dots(ii) \quad [\text{Using (i)}]$$

Differentiating (i) w.r.t.  $x$ , we get

$$\begin{aligned} \frac{dS}{dx} &= 1 \cdot \sqrt{100 - x^2} + (x + 10) \cdot \frac{1}{2\sqrt{100 - x^2}} \cdot (-2x) \\ &= \frac{100 - x^2 - x^2 - 10x}{\sqrt{100 - x^2}} = \frac{-2(x^2 + 5x - 50)}{\sqrt{100 - x^2}} \\ &= \frac{-2(x + 10)(x - 5)}{\sqrt{100 - x^2}} \end{aligned}$$

For max. or min. value of  $S$ ,  $\frac{dS}{dx} = 0$

$$\Rightarrow (x + 10)(x - 5) = 0 \Rightarrow x = 5$$

(Reject  $x = -10$  as  $x \neq 0$ )

For this value of  $x$ ,  $\frac{dS}{dx} < 0$   
 $\therefore S$  is maximum at  $x = 5$ .

From (ii), the max. value of  $S = (5 + 10) \cdot \sqrt{100 - 5^2}$   
 $= 15\sqrt{75} = 75\sqrt{3}$  sq.cm.

29. Let  $E_i (i = 1, 2, 3)$  be the event of drawing a queen in the  $i^{\text{th}}$  draw. Let  $X$  denote the discrete random variable "Number of Queens" in 3 draws one by one with replacement.

Here  $X = 0, 1, 2, 3$

Now  $P(E_i) = \frac{4}{52}$ ;  $P(\bar{E}_i) = \frac{48}{52} (i = 1, 2, 3)$

$$\begin{aligned}\therefore P(X = 0) &= P(\bar{E}_1 \bar{E}_2 \bar{E}_3) = P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3) \\ &= \left(\frac{48}{52}\right)^3 = \left(\frac{12}{13}\right)^3 = \frac{1728}{2197} \\ P(X = 1) &= P(E_1 \bar{E}_2 \bar{E}_3 \text{ or } \bar{E}_1 E_2 \bar{E}_3 \text{ or } \bar{E}_1 \bar{E}_2 E_3) \\ &= P(E_1 \bar{E}_2 \bar{E}_3) + P(\bar{E}_1 E_2 \bar{E}_3) + P(\bar{E}_1 \bar{E}_2 E_3) \\ &= P(E_1)P(\bar{E}_2)P(\bar{E}_3) + P(\bar{E}_1)P(E_2)P(\bar{E}_3) \\ &\quad + P(\bar{E}_1)P(\bar{E}_2)P(E_3) \\ &= 3 \times \frac{4}{52} \times \left(\frac{48}{52}\right)^2 = 3 \times \frac{1}{13} \times \left(\frac{12}{13}\right)^2 = \frac{432}{2197}\end{aligned}$$

$$\begin{aligned}P(X = 2) &= P(E_1 E_2 \bar{E}_3 \text{ or } E_1 \bar{E}_2 E_3 \text{ or } \bar{E}_1 E_2 E_3) \\ &= P(E_1 E_2 \bar{E}_3) + P(E_1 \bar{E}_2 E_3) + P(\bar{E}_1 E_2 E_3) \\ &= P(E_1)P(E_2)P(\bar{E}_3) + P(E_1)P(\bar{E}_2)P(E_3) \\ &\quad + P(\bar{E}_1)P(E_2)P(E_3) \\ &= 3 \times \left(\frac{4}{52}\right)^2 \times \left(\frac{48}{52}\right) = 3 \times \left(\frac{1}{13}\right)^2 \times \frac{12}{13} = \frac{36}{2197}\end{aligned}$$

$$\begin{aligned}P(X = 3) &= P(E_1 E_2 E_3) = P(E_1)P(E_2)P(E_3) \\ &= \left(\frac{4}{52}\right)^3 = \frac{1}{2197}\end{aligned}$$

$\therefore$  The required probability distribution of  $X$  is

$X$	0	1	2	3
$P(X)$	$\frac{1728}{2197}$	$\frac{432}{2197}$	$\frac{36}{2197}$	$\frac{1}{2197}$

OR

Let  $E$  = the event that  $A$  wins a game against  $B$ .  
 Let occurrence of the event  $E$  be called a success and  $X$  denote the number of successes.  
 Let  $E_1$  = the event that  $A$  wins 'a best of 3 games' match.

$A$  will win the best of 3 games match if he wins in 2 or 3 games

$E_2$  = the event that  $A$  wins 'a best of 5 games' match.

$A$  will win a best of 5 games match if he wins in 3 or 4 or 5 games.

$$\begin{aligned}\text{Now } P(E_1) &= P(X = 2 \text{ or } X = 3) = P(X = 2) + P(X = 3) \\ &= {}^3C_2 p^2 q + {}^3C_3 p^3 = 3(0.4)^2(0.6) + (0.4)^3 \\ &= (0.4)^2[1.8 + 0.4] = (0.4)^2(2.2) = 0.352\end{aligned}$$

$$\begin{aligned}P(E_2) &= P(X = 3 \text{ or } X = 4 \text{ or } X = 5) \\ &= P(X = 3) + P(X = 4) + P(X = 5) \\ &= {}^5C_3 p^3 q^2 + {}^5C_4 p^4 q + {}^5C_5 p^5 \\ &= (10p^3 q^2 + 5p^4 q + p^5) = p^3(10q^2 + 5pq + p^2) \\ &= (0.4)^3 \times [10 \times (0.6)^2 + 5 \times (0.4) \times (0.6) + (0.4)^2] \\ &= (0.064) \times (3.6 + 1.2 + 0.16) \\ &= 0.064 \times 4.96 = 0.317\end{aligned}$$

Since  $P(E_1) > P(E_2)$ , hence  $A$  should choose the first option.

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# MPP-8 MONTHLY Practice Problems

Class XII



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## VECTORS & THREE DIMENSIONAL GEOMETRY

Total Marks : 80

Time Taken : 60 Min.

### Only One Option Correct Type

- If the volume of parallelopiped with  $\vec{a} \times \vec{b}$ ,  $\vec{b} \times \vec{c}$ ,  $\vec{c} \times \vec{a}$  as co-terminous edges is 9 cu. units, then the volume of the parallelopiped with  $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})$ ,  $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$ ,  $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$  as co-terminous edges is  
(a) 9 (b) 729 (c) 81 (d) 27
- If the plane  $2ax - 3ay + 4az + 6 = 0$  passes through the midpoint of the line joining the centres of the spheres,  $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$  and  $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ , then  $a$  equals  
(a) 1 (b) -1 (c) 2 (d) -2
- If  $\vec{p} \times \vec{q} = \vec{r}$  and  $\vec{q} \times \vec{r} = \vec{p}$ , then  
(a)  $r = 1, p = q$  (b)  $p = 1, q = 1$   
(c)  $r = 2p, q = 2$  (d)  $q = 1, p = r$
- Let the line  $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$  lie in the plane  $x + 3y - \alpha z + \beta = 0$ . Then  $(\alpha, \beta)$  equals  
(a)  $(-6, 7)$  (b)  $(5, -15)$   
(c)  $(-5, 5)$  (d)  $(6, -17)$
- Let  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ ,  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  and  $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$  be three non-zero vectors such that  $\vec{c}$  is a unit vector perpendicular to both the vectors  $\vec{a}$  and  $\vec{b}$  and if the angles between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{6}$ , then  $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$  is equal to  
(a) 1  
(b)  $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

$$(c) \frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$$

(d) none of these

- If the angle between the line  $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$  and

the plane  $x + 2y + 3z = 4$  is  $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ , then  $\lambda$  equals

(a) 2/5 (b) 5/3 (c) 2/3 (d) 3/2

### One or More Than One Option(s) Correct Type

- If a vector  $\vec{r}$  satisfies the equation  $\vec{r} \times (\hat{i} + 2\hat{j} + \hat{k}) = \hat{i} - \hat{k}$ , then  $\vec{r}$  may be equal to  
(a)  $\hat{i} + 3\hat{j} + \hat{k}$  (b)  $3\hat{i} + 7\hat{j} + 3\hat{k}$   
(c)  $\hat{j} + t(\hat{i} + 2\hat{j} + \hat{k})$ , where  $t$  is any scalar  
(d)  $\hat{i} + (t+3)\hat{j} + \hat{k}$ , where  $t$  is any scalar
- If  $OABC$  is a tetrahedron such that  $OA^2 + BC^2 = OB^2 + CA^2 = OC^2 + AB^2$ , then  
(a)  $OA \perp BC$  (b)  $OB \perp CA$   
(c)  $OC \perp AB$  (d)  $AB \perp BC$
- If  $\vec{r}$  is equally inclined to the co-ordinate axes and magnitude of  $\vec{r} = 6$  then  $\vec{r}$  equals  
(a)  $\sqrt{3}(\hat{k} + \hat{i} + \hat{j})$  (b)  $\frac{2(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$   
(c)  $\frac{6(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$  (d)  $-2\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$
- If the unit vectors  $\vec{a}$  and  $\vec{b}$  are inclined at an angle  $2\theta$  and  $|\vec{a} - \vec{b}| < 1$ , then if  $0 \leq \theta \leq \pi$ ,  $\theta$  lies in the interval  
(a)  $[0, \pi/6]$  (b)  $(5\pi/6, \pi]$   
(c)  $[\pi/6, \pi/2]$  (d)  $[\pi/2, 5\pi/6]$

11. Let the unit vectors  $\vec{A}$  and  $\vec{B}$  be perpendicular and the unit vector  $\vec{C}$  be inclined at an angle  $\theta$  to both  $\vec{A}$  and  $\vec{B}$ . If  $\vec{C} = \alpha \vec{A} + \beta \vec{B} + \gamma(\vec{A} \times \vec{B})$  then

- (a)  $\alpha = \beta$  (b)  $\gamma^2 = 1 - 2\alpha^2$   
(c)  $\gamma^2 = -\cos 2\theta$  (d)  $\beta^2 = \frac{1 + \cos 2\theta}{2}$

12. The equation of a sphere which passes through (1, 0, 0), (0, 1, 0) and (0, 0, 1) and whose centre lies on the curve  $4xy = 1$  is

- (a)  $x^2 + y^2 + z^2 - x - y - z = 0$   
(b)  $x^2 + y^2 + z^2 + x + y + z - 2 = 0$   
(c)  $x^2 + y^2 + z^2 + x + y + z = 0$   
(d)  $x^2 + y^2 + z^2 - x - y - z - 2 = 0$

13. The points in which the line  $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z+3}{1}$

cuts the surface  $x^2 + y^2 + z^2 - 20 = 0$  are

- (a) (0, 2, 4) (b) (0, 2, -4)  
(c) (4, 2, 0) (d) (0, -2, -4)

#### Comprehension Type

If  $\vec{a}, \vec{b}, \vec{c}$  are three given non-coplanar vectors and  $\vec{r}$  be any arbitrary vector in space, where

$$\Delta_1 = \begin{vmatrix} \vec{r} \cdot \vec{a} & \vec{b} \cdot \vec{a} & \vec{c} \cdot \vec{a} \\ \vec{r} \cdot \vec{b} & \vec{b} \cdot \vec{b} & \vec{c} \cdot \vec{b} \\ \vec{r} \cdot \vec{c} & \vec{b} \cdot \vec{c} & \vec{c} \cdot \vec{c} \end{vmatrix}, \Delta_2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{r} \cdot \vec{a} & \vec{c} \cdot \vec{a} \\ \vec{a} \cdot \vec{b} & \vec{r} \cdot \vec{b} & \vec{c} \cdot \vec{b} \\ \vec{a} \cdot \vec{c} & \vec{r} \cdot \vec{c} & \vec{c} \cdot \vec{c} \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{b} \cdot \vec{a} & \vec{r} \cdot \vec{a} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} & \vec{r} \cdot \vec{b} \\ \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} & \vec{r} \cdot \vec{c} \end{vmatrix} \text{ and } \Delta = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{b} \cdot \vec{a} & \vec{c} \cdot \vec{a} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} & \vec{c} \cdot \vec{b} \\ \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} & \vec{c} \cdot \vec{c} \end{vmatrix}, \text{ then}$$

14. If vector  $\vec{r}$  is expressible as,  $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$ , then

- (a)  $\vec{a} = \frac{\vec{a} \cdot \vec{a}}{[\vec{a} \vec{b} \vec{c}]}(\vec{b} \times \vec{c}) + \frac{\vec{a} \cdot \vec{b}}{[\vec{a} \vec{b} \vec{c}]}(\vec{c} \times \vec{a}) + \frac{\vec{c} \cdot \vec{a}}{[\vec{a} \vec{b} \vec{c}]}(\vec{a} \times \vec{b})$   
(b)  $\vec{a} = \vec{a} \cdot \vec{a}(\vec{b} \times \vec{c}) + \vec{a} \cdot \vec{b}(\vec{c} \times \vec{a}) + \vec{c} \cdot \vec{a}(\vec{a} \times \vec{b})$   
(c)  $\vec{a} = [\vec{a} \vec{b} \vec{c}](\vec{b} \times \vec{c}) + [\vec{a} \vec{b} \vec{c}](\vec{c} \times \vec{a}) + [\vec{a} \vec{b} \vec{c}](\vec{a} \times \vec{b})$   
(d) None of these.

15. The value for  $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a} \cdot \vec{p} & \vec{b} \cdot \vec{p} & \vec{c} \cdot \vec{p} \\ \vec{a} \cdot \vec{q} & \vec{b} \cdot \vec{q} & \vec{c} \cdot \vec{q} \end{vmatrix}$ , is

- (a)  $(\vec{p} \times \vec{q})[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}]$   
(b)  $2(\vec{p} \times \vec{q})[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}]$   
(c)  $4(\vec{p} \times \vec{q})[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}]$   
(d)  $(\vec{p} \times \vec{q})\sqrt{[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}]}$ .

#### Matrix Match Type

16. Match the following:

Column I		Column II	
P.	A line is perpendicular to $x + 2y + 2z = 0$ and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	1.	$\frac{\sqrt{5}}{3}$
Q.	A plane passes through (1, -2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$ . The distance of the plane from the point (1, 2, 2) is	2.	$2\sqrt{2}$
R.	Point $(\alpha, \beta, \gamma)$ lies on the plane $x + y + z = 2$ . Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ , $\hat{k} \times (\hat{k} \times \vec{a}) = \vec{0}$ , then $\gamma =$	3.	2

- |       |   |   |
|-------|---|---|
| P     | Q | R |
| (a) 2 | 1 | 3 |
| (b) 3 | 4 | 2 |
| (c) 3 | 2 | 4 |
| (d) 1 | 2 | 3 |

#### Integer Answer Type

17. The line passes through the points (5, 1, a) and

(3, b, 1) crosses the yz-plane at the point  $(0, \frac{17}{2}, -\frac{13}{2})$ . then  $a - b =$

18. If  $[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}] = \lambda[\vec{a} \vec{b} \vec{c}]^2$ , then  $\lambda$  is equal to

19. The plane  $x + 2y - z = 4$  cuts the sphere

$x^2 + y^2 + z^2 - x + z - 2 = 0$  in a circle of radius

20. Let  $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}, \vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$ . If  $\hat{n}$  is a unit vector such that  $\vec{u} \cdot \hat{n} = 0$  and  $\vec{v} \cdot \hat{n} = 0$  then  $|\vec{w} \cdot \hat{n}|$  equals

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## SELF CHECK

No. of questions attempted .....  
No. of questions correct .....  
Marks scored in percentage .....

### Check your score! If your score is

- |        |                    |  |
|--------|--------------------|--|
| > 90%  | EXCELLENT WORK !   | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK !        | You can score good in the final exam.                      |
| 74-60% | SATISFACTORY !     | You need to score more next time.                          |
| < 60%  | NOT SATISFACTORY ! | Revise thoroughly and strengthen your concepts.            |

# MATHS MUSING

## SOLUTION SET-169

1. (c) :  $z_1, z_2, z_3$  are roots  $\Rightarrow \Sigma z_1 = -p, \Sigma z_1 z_2 = q$

$$p^2 = \Sigma z_1^2 + 2q$$

For equilateral triangle,  $\Sigma z_1^2 = \Sigma z_1 z_2$

$$\therefore p^2 = q + 2q = 3q$$

$$2. (b) : \frac{1}{a} + \frac{1}{b} = \frac{1}{2013}$$

$$\Rightarrow (a - 2013)(b - 2013) = 2013^2$$

$$= 3^2 \cdot 11^2 \cdot 61^2$$

The number of all pairs is the number of divisors of  $3^2 \cdot 11^2 \cdot 61^2$ , which is  $3^3 = 27$ .

Number of pairs  $(a, b)$ ,  $a < b$  is  $\frac{27-1}{2} = 13$

3. (a) :  $x^2 - ix - 1 = 0$ . Solving,

$$x = \text{cis } \frac{\pi}{6} \text{ and } x = \text{cis } \frac{5\pi}{6}$$

$$x^{2013} = \text{cis } (2013) \frac{\pi}{6} \text{ and } x^{2013} = \text{cis } (10065) \frac{\pi}{6}$$

$$\Rightarrow x^{2013} = -i, x^{2013} + x^{-2013} = -i + i = 0$$

$$4. (c) : \int_0^{100} \{\sqrt{x}\} dx = \int_0^{100} \sqrt{x} dx - \sum_{r=0}^9 \int_{r^2}^{(r+1)^2} r dx$$

$$= \frac{2000}{3} - \sum_{r=1}^9 (2r^2 + r) = \frac{2000}{3} - 615 = 51 \frac{2}{3}$$

5. (b) : The parabola is open upwards with vertex  $(2, -1)$ . The focus is  $(2, -1 + 1) = (2, 0)$ .

The reflected ray passes through the focus  $(2, 0)$ .

6. (a, d) : The equation represents a pair of planes.

$$\Rightarrow \begin{vmatrix} a & \frac{3}{2} & \frac{1}{2} \\ \frac{3}{2} & 1 & 1 \\ \frac{1}{2} & 1 & b \end{vmatrix} = 0 \Rightarrow 4ab - 4a - 9b + 5 = 0 \quad \dots(i)$$

$$\text{For perpendicular planes } a + 1 + b = 0 \quad \dots(ii)$$

Eliminating  $b$  from (i) and (ii), we get

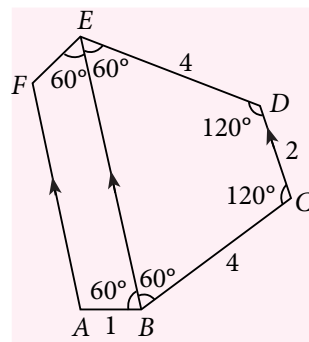
$$4a^2 - a - 14 = 0 \Rightarrow a = 2, -\frac{7}{4}$$

7. (c) :  $ABEF$  and  $BCDE$  are trapeziums.

$$BE = 2 + 2 \cdot 4 \cos 60^\circ = 6$$

$$AF = BE - 2 \cos 60^\circ = 6 - 1 = 5$$

$$\text{Perimeter} = 1 + 4 + 2 + 4 + 1 + 5 = 17$$



8. (d) : Area = Area of  $BCDE$  + Area of  $ABEF$

$$= \frac{1}{2} (6 + 2) 4 \sin 60^\circ + \frac{1}{2} (6 + 5) \sin 60^\circ$$

$$= 8\sqrt{3} + \frac{11}{4} \sqrt{3} = \frac{43}{4} \sqrt{3}$$

9. (4) :  $N$  is the coefficient of  $x^{30}$  in

$$(1 + x + x^2 + \dots + x^{10})^3 (1 + x + x^2 + \dots + x^{20})$$

$$(1 - x^{11})^3 (1 - x^{21})(1 - x)^{-4}$$

$$= (1 - 3x^{11} + 3x^{22} - x^{33} \dots) \left( 1 + \binom{4}{1}x + \binom{5}{2}x^2 + \dots \right)$$

$$\text{which is } \binom{33}{30} - 3 \binom{22}{19} + 3 \binom{11}{8} - \binom{12}{9}$$

$$= \binom{33}{3} - 3 \binom{22}{3} + 3 \binom{11}{3} - \binom{12}{3} = 1111,$$

with digit sum 4.

10. (a)-(s, t); (b)-(p), (c)-(r), (d)-(r)

$$(a) f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0} x^{\alpha-1} \sin\left(\frac{1}{x}\right) = 0$$

if  $\alpha > 1$

$$x \neq 0 \Rightarrow f'(x) = \alpha x^{\alpha-1} \sin\left(\frac{1}{x}\right) - x^{\alpha-2} \cos\left(\frac{1}{x}\right)$$

$$\lim_{x \rightarrow 0} f'(x) = 0 \text{ if } \alpha > 2 \quad \therefore \alpha = 3, 4.$$

$$(b) y(1) = 2 \Rightarrow 2 = a + b + c, y(0) = 0, y'(0) = 1$$

$$\Rightarrow c = 0, b = 1, a = 1 \quad \therefore y = x^2 + x, y(-1) = 0$$

$$(c) f(x) = \sin x(1 + a), \text{ where } a = \int_0^{\pi/2} \cos t dt = 1$$

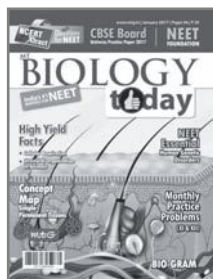
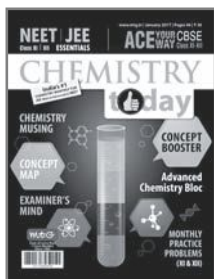
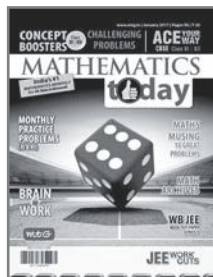
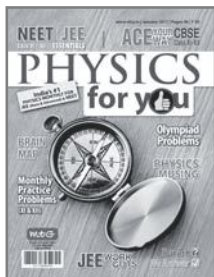
$$\therefore f(x) = 2 \sin x, \int_0^{\pi/2} f(x) dx = 2.$$

(d) The hyperbolas  $\frac{x^2}{2} - y^2 = 1$  and  $\frac{y^2}{2} - x^2 = 1$  have

common tangents with slopes  $\pm 1$ . The four tangents are  $x + y \pm 1 = 0, x - y \pm 1 = 0$ . They form a square of area  $\sqrt{2} \times \sqrt{2} = 2$



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